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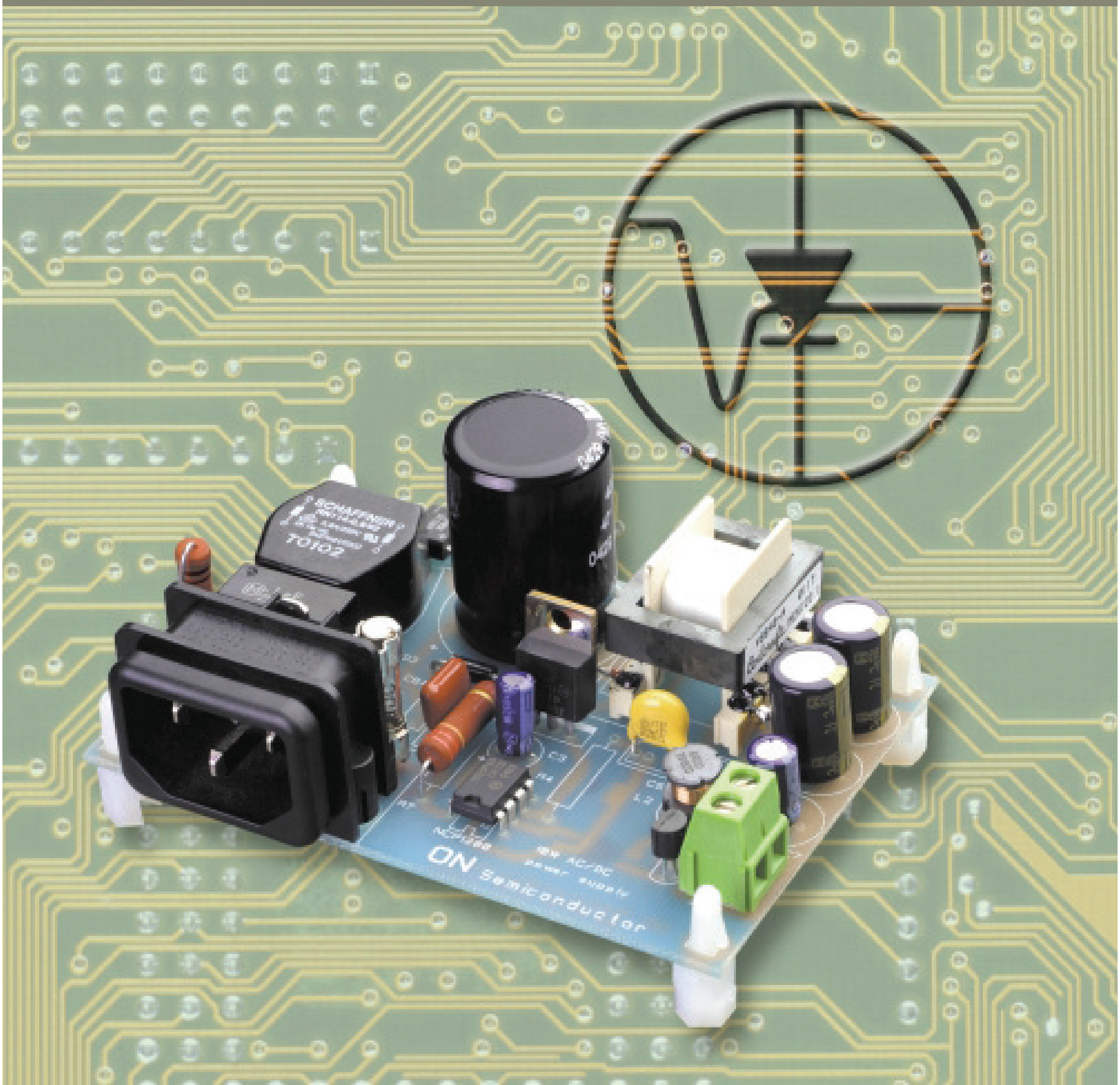
Rectifier Applications Handbook



Chapter 5

Basic Single–Phase Rectifying Circuits

Basic Single-Phase Rectifying Circuits

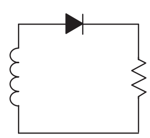
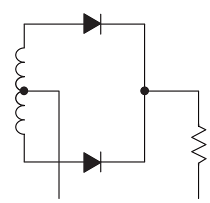
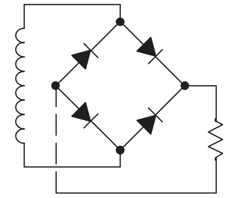
In this chapter, basic characteristics of single–phase circuits with resistive loads are discussed. Polyphase circuits are treated in Chapter 6; filter design follows in Chapter 7. Results of all the calculations in this chapter are presented in Table 16.

Basic Operation

When considering any rectifying circuit, a designer desires to know the magnitude of the direct voltage and current, the regulation of the load voltage, and the efficiency to be expected from the rectifying process. All these values depend upon a number of variables–such as the type of circuit, the constants of the supply, the characteristics of the rectifying unit, and the nature of the load–which complicate the analytical solution. However, certain simplifying assumptions idealize the circuit so that a useful analysis can be made.

The simplest circuit for rectifying single–phase alternating torrent gives half–wave rectification. Such a circuit is indicated in Figure 86(a), where the bold–faced arrow represents the rectifying unit and the direction of conventional current flow. Assume (1) an ideal AC source without resistance, (2) the impressed emf is a pure sine wave, (3) the rectifying unit has zero resistance in the forward direction of current and infinite resistance in the reverse direction, and (4) the load is purely resistive. With these assumptions, let a sine wave alternating voltage as shown in Figure 86(b) be impressed across the input to the rectifying circuit. The output is a rectified half–wave of current as indicated in Figure 86(c), which is of a sine form, since $i = (V_M/R_L) \sin \omega t$. During the second half of the cycle the rectifier blocks current. The current flowing through the load resistance R_L produces an iR_L voltage drop which has a half sine waveform. The voltage waveform across the rectifier is shown by Figure 86(e).

Table 16. Characteristics of Basic Single–Phase Rectifier Circuit with Resistive Loads

	Half–Wave	Full–Wave Center–Tap	Full–Wave Bridge
Rectifier Circuit Connection			
Load Voltage and Current Waveshape Characteristic			
Diode Average Current $I_{F(AV)}/I_{L(DC)}$	1.00	0.50	0.50
Diode Peak Current $I_{FM}/I_{F(AV)}$	3.14	3.14	3.14
Form Factor of Diode $I_{F(RMS)}/I_{(DC)}$	1.57	1.57	1.57
Diode RMS Current $I_{F(RMS)}/I_{L(DC)}$	1.57	0.785	0.785
RMS Input Voltage Per Transformer Leg $V_i/V_{L(DC)}$	2.22	1.11	1.11
Peak Inverse Voltage $V_{RRM}/V_{L(DC)}$	3.14	3.14	1.57
Transformer Primary Rating VA/P_{DC}	3.49	1.23	1.23
Transformer Secondary Rating VA/P_{DC}	3.49	1.75	1.23
Total RMS Ripple, %	121	48.2	48.2
Lowest Ripple Frequency, f_r/f_i	1	2	2
Rectification Ratio (Conversion Efficiency), %	40.6	81.2	81.2

1. $P = I_L^2 R_L$ $V_L = I_L R_L$

Current Relationships

Since the function of rectification is to convert alternating current to direct current, the equivalent value of the direct current output is of primary interest. Its value is measured by the dc ammeter placed in the circuit of Figure 86(a).

This dc value is the average value of th6 instantaneous rectified current over one cycle or a period corresponding to 2π radians. Average values can be determined by a graphical and arithmetic process by measuring the instantaneous value of current at a series of equally spaced points along the time axis and then dividing the sum of these values by the total number of points in a cycle. A more accurate solution may be obtained by measuring the area under the rectified current pulse and dividing by 2π. The more direct solution is to apply calculus to the problem, thus:

$$I_{DC} = I_{AV} = \frac{1}{2\pi}\int_0^{\pi} I_M \sin\omega t d(\omega t) \qquad (5.1)$$

Solving,

$$I_{DC} = \frac{2I_M}{2\pi} = 0.318\ I_M \qquad (5.2)$$

An ac ammeter inserted in the rectifying circuit of Figure 86(a) gives a different reading from the dc meter first considered. This difference arises because the ac meter registers effective or root–mean–square (rms) values [1] rather than average values. The heating or effective value of a current varies as the square of the instantaneous current. Hence, to determine the effective value of the rectified current graphically, it is first necessary to plot squared values of the instantaneous current as suggested by the dotted i^2 curve on Figure 86(c). The effective area under the dotted i^2 curve may be obtained by the graphical point method, or by calculus:

$$\text{Effective area} = \int_0^{\pi} I_M^2 \sin^2\omega t d(\omega t)$$

$$= I_M^2\left[\frac{\omega t}{2} - \frac{\sin 2\omega t}{4}\right]_0^{\pi} = \frac{I_M^2\pi}{2} \qquad (5.3)$$

and the effective or rms current,

$$\sqrt{\frac{\text{effective area}}{2\pi}} = 0.5\ I_M \qquad (5.4)$$

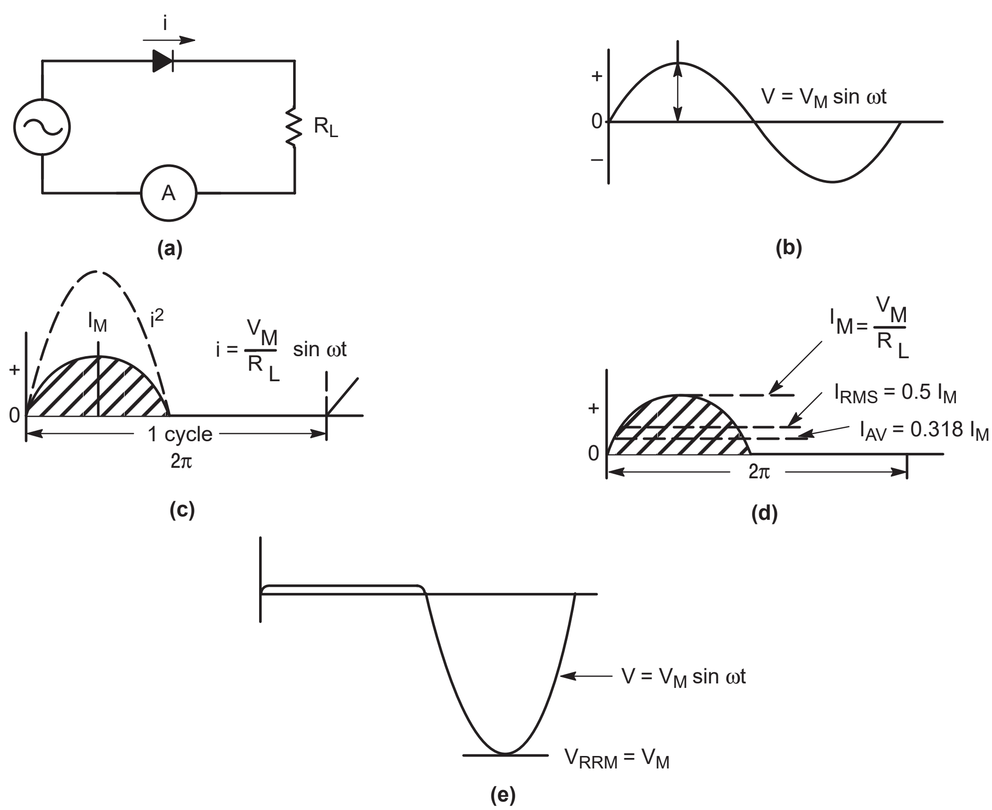


Figure 86. Circuit and Wave Shapes for Half–Wave Rectification: (a) Half–Wave Circuit; (b) Input Waveform; (c) Current Waveform; (d) Current Waveform with Pertinent Points Indicated; (e) Rectifier Voltage Waveform

The average and effective values of rectified current for the half–wave circuit are indicated in Figure 86. The integration of Equations 5.1 and 5.3 follow the standard procedures for obtaining the average and effective, values of periodic functions.:

Half–wave circuits are not used with transformers unless current requirements are small, because the dc component of output current must flow through the transformer. The dc component causes core magnetization and high core losses result. However, the half–wave circuit finds limited use in low–current, direct–line rectification.

Two types of circuits are used for full–wave, single–phase rectification. One circuit uses a transformer with a mid–tap in the secondary winding, as shown in Figure 87(a). The other uses a bridge configuration, shown in Figure 87(b), which requires two extra rectifier diodes, but the secondary requires only half as much winding. Performance is similar except that the bridge diodes are subjected to only half the peak inverse voltage of the center–tap circuit (to be discussed later). Since both half–waves of current pass through the transformer (dividing in the secondary of the center–tap circuit), there is no dc component of flux in the transformer core to increase core losses.

Using the same assumptions made for the preceding half–wave rectifier circuit, the relation between the input and output sides of either full–wave rectifier circuit may be readily calculated. Since both halves of the cycle are rectified, the current and voltage on the input side are normal effective values; rms values on the output side are the same as for a sine wave while the dc or average values are twice that of a half–wave circuit. The relationships are shown in Figure 87(d).

Form Factor

Rectifier circuit efficiencies can be related to a quantity termed form factor (fl. It is the ratio of the heating component of a wave to the dc component:

$$F = \frac{I_{(RMS)}}{I_{(AV)}} \qquad (5.5)$$

Substituting the values from Equations 5.2 and 5.4 into Equation 5.5, the form factor for a half–wave circuit is 1.57. The form factor is the same for each rectifier element in the full–wave systems because each side conducts on opposite half cycles. However, the form factor of the output current of a full–wave circuit is much better. It is found from the values shown on Figure 87; i.e., F = (0.707)1(6.636) = 1.11.

Form factor takes on significance when rectifiers must handle high peak currents at low duty cycles, since the power losses in the diodes and transformers are much higher than encountered with sine wave pulses. Applications where high form factors are the rule occur when capacitive input filters are used, when batteries are being charged, and when rectifiers are used with SCRs in phase control circuitry.

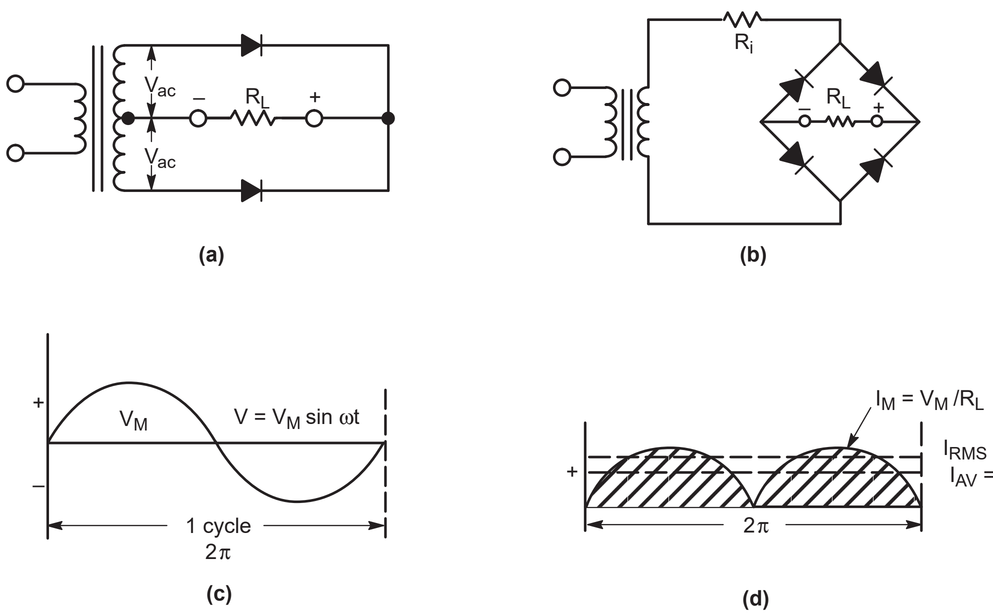


Figure 87. Circuit and Waveshapes for Full–Wave Rectification: (a) Center–Tap Circuit; (b) Bridge Circuit; (c) Input Waveform; (d) Output Waveform

Utilization Factor

Because of the waveforms involved in rectifier circuits, transformers are not used as efficiently as when they handle pure sinusoidal waveforms. A measure of rectifier circuit merit is the utilization factor (UP), defined as the ratio of the dc output power to the transformer volt–ampere rating required by the primary and/or the secondary. For single–phase circuits with resistive loads, the UF can be found by using the relationships between rms and average current. As defined,

$$U_F = \frac{I_{AV}V_{AV}}{I_{RMS}V_{RMS}} \tag{5.6}$$

For the half–wave circuit:

$$U_F = (0.318\ I_M)\ V_{DC}/(0.51\ I_M)(2.22\ V_{DC}) = 0.286 \tag{5.7}$$

For a full–wave circuit, transformer utilization is much improved because conduction is continuous. In the center–tap circuit, although both the secondary windings are present, only one is used at a time. The utilization factor for the secondary is found as

$$V_F = \frac{(0.318\ I_M)(2)V_{DC}}{2(0.5\ I_M)(1.11)V_{DC}} = 0.572$$

In the primary, the whole winding is naturally in continuous use; the input power is, assuming a 1:1 winding for simplicity, $(\sqrt{2}/2)I_M V_{i(RMS)}$.

Therefore,

$$V_F = \frac{(0.318\ I_M)(2)V_{DC}}{(\sqrt{2}/2)I_M(1.11)V_{DC}} = 0.812$$

The bridge rectifier circuit has the same utilization factor as the primary of the full–wave center–tap circuit. The UP of the single–phase bridge is quite high and is only exceeded by certain polyphase circuits.

However, utilization factor does not tell the whole story in the case of a half–wave circuit since only one half of the sine wave of current is passed, and the windings carry a dc component of current which magnetizes the iron core and increases the core losses. As a result, half–wave rectifiers are only practical for use with a transformer when the current requirement is very small.

Harmonics

When the current in a transformer winding does not have a sinusoidal waveform, harmonic currents are present. The harmonic content can be found by a Fourier analysis of the waveform or a spectrum analysis of current in an operating circuit. For half sine waves the percentage of each harmonic with respect to the fundamental is given in Table 17:

Table 17. Harmonic Percentages of a Half Sine Wave

Harmonic	2nd	3rd	4th	5th	6th
%	21.2	0	4.2	0	1.8

Harmonics cause transformer core loss and hysteresis loss to be higher than with single–frequency operation because these lasses increase with frequency. The extra loss caused by the second harmonic in the resistive loaded rectifier circuit is not high compared to other losses in the transformer and is usually neglected. ‘With other loads, losses caused by harmonics can be appreciable and not only complicate transformer design but also cause trouble with the electrical distribution system.

Ripple Factor

The rectified voltage and current output consists of a series of unidirectional waves or ripples. For some applications these variations are not objectionable, but for others they must be smoothed out by filters. For all cases, the relative magnitude of the ripple is important in the comparison of rectifying circuits. The comparison is made in terms of ripple factor. Ripple factor is the ratio of effective value of the alternating components of the rectified voltage or current to the average value. In equation form ripple factor is

$$r_f = \frac{\text{effective rectified ac load component } I'_{rms}}{\text{average load current } (I_{DC})} \tag{5.7}$$

Percent ripple is a term used interchangeably with ripple factor and is simply the ripple factor expressed in percent (r_f X 100).

The various components of current associated with ripple factor are illustrated in Figure 88. Figure 88(a) shows the single current pulse of half–wave rectification, and Figure 88(b) shows the splitting of the pulse into a dc component and a ripple component. Figures 88(c) and 88(d) give an actual separation of the components, and Figure 88(e) illustrates the components for full–wave rectification.

In accordance with the preceding definition, the ripple factor is the ratio of the effective current represented by I’ of Figure 88(c) to the dc component shown in Figure 88(d). This ratio may be computed by following the preceding form of calculation. Thus, the instantaneous ac ripple component i' may be represented as

$$i' = i - I_{DC}, \tag{5.8}$$

and the total rms value of the ripple component I'_{rms} is

$$I'_{rms} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi}(i - I_{DC})^2\, d\theta}$$

$$I'_{rms} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi}(i^2 - 2I_{DC} + i^2_{DC})\, d\theta} \tag{5.9}$$

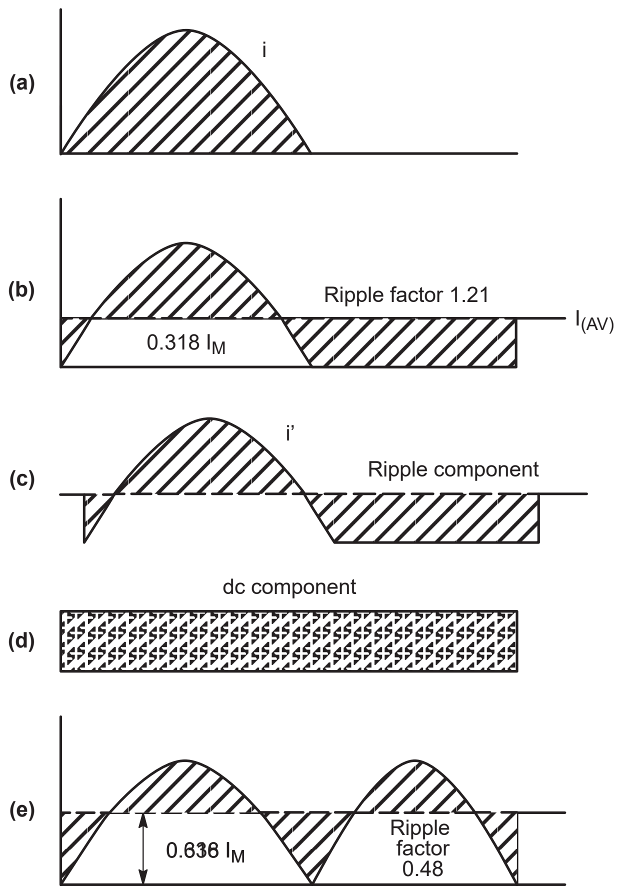


Figure 88. Illustration of Ripple Factors for Half–Wave and Full–Wave Circuits

The first term in this expression is the rms value of the total current $I_{(RMS)}$. The integral of the idθ part of the second term integrates to the average value $I_{(DC)}$, and the last term is simply $I^2_{(DC)}$ after the limits are applied. Thus:

$$I'_{rms} = \sqrt{I^2_{RMS} - 2I^2_{DC} + I^2_{DC}} \quad (5.10)$$

By combining Equations 5.10 and 5.5, the ripple factor may be expressed as

$$r_f = \sqrt{F^2 - 1} \quad (5.11)$$

(F is the form factor of the output current, not the diode current.) Substitution of values from Figures 86 and 87 into Equation 5.11 gives

$$\text{Half–wave } F = \frac{0.5I_M}{0.318I_M} = 1.57$$

$$r_f = \sqrt{1.57^2 - 1} = 1.21$$

$$\text{Full–wave } F = \frac{0.707I_M}{0.636I_M} = 1.11$$

$$r_f = \sqrt{1.11^2 - 1} = 0.482$$

Rectification Ratios

The preceding discussion of ripple factor leads to the idea that the heating losses (I^2R) which occur in the various parts of the complete rectifier circuit are increased by the irregular waveforms of current that are inherent in the rectifying process. This loss may be illustrated by comparing the heating loss caused by an ideal direct current to that resulting from the actual current waves of Figures 88(a), 88(b), and 88(c). The ratio of dc power in the load to ac rms power in the load is termed the rectification ratio, Ó and is written as

$$\sigma = \frac{I^2_{(DC)}R_L}{I^2_{(RMS)}R_L} \quad (5.12)$$

Substitution of the appropriate current values in terms of Im yields:

Ó = .406 (half–wave),

Ó = .812 (full–wave),

The rectification ratio is sometimes called the conversion efficiency. This term is somewhat misleading since the overall power efficiency for the assumed conditions (zero losses) must be 100%. The real significance of the rectification ratio is that it gives a qualitative indication of the increased heat losses that occur wherever a pulsating current flows through resistance elements. A second method of expressing the increased heat losses is by the current form factor, discussed earlier, which is the ratio of the root–mean–square to the average value.

Voltage Relationships

The input voltage required to achieve a given dc output voltage (V_{DC}) is a necessary piece of design information. The output voltage (V_{DC}) is I_{DC} R_L and Equation 5.2 states that $I_{DC} = 0.318\ I_M$ for the half–wave circuit. The peak current (I_M) is V_M/R_L and the rms input voltage $V_i = V_m/\sqrt{2}$

Combining these relationships yields:

$$V_i = 2.22\ V_{DC} \qquad \text{(half–wave)}$$

The full–wave circuit produces twice the dc level as the half–wave circuit for a given input voltage per transformer leg. Consequently,

$$V_i = 1.11\ V_{DC} \qquad \text{(full–wave)}$$

Another voltage of importance is the maximum voltage which the rectifier must block when it is not conducting. It is called the peak inverse voltage, or, in rectifier parlance, V_{RRM}. V_{RRM} is the sum of the peak input voltage and the rectifier peak output voltage at the same instant in time.

In the half–wave circuit of Figure 86, the output voltage is zero when the input voltage reaches its negative peak; therefore,

$$V_{RRM} = V_M = (\sqrt{2})(2.22\ V_{DC}) = 3.14\ V_{DC}. \qquad \text{(half–wave)}$$

In the full–wave center–tap circuit, the output voltage is maximum at V_M, because of rectifier conduction on one side, when the other side must block V_M. Therefore, $V_{RRM} = 2\ V_M$, but now the rms input voltage, V_M is 1.11 V_{DC}, consequently

$$V_{RRM} = (\sqrt{2})(2)(1.11)\ V_{DC} = 3.14\ V_{DC}.$$

(full–wave center–tap)

For the bridge, the blocking rectifiers are across the output voltage; therefore, the maximum reverse voltage is V_M. In this case,

$$V_{RMM} = (\sqrt{2})(1.11)\ V_{DC} = 1.57\ V_{DC}.$$

(full–wave bridge)

Circuit Variations

Many applications require equal positive and negative power supply voltages. Circuits shown in Figure 89 are commonly used to provide dual output voltages. These circuits are simply combinations of the standard half–wave and center–tapped full–wave circuits previously discussed. Chapter 2 illustrates how a rectifier bridge assembly may be properly handled thermally in the circuit of Figure 89(b). If the loads are balanced, the circuit of Figure 89(a) does not have a dc component in the transformer core, thereby overcoming the major objection to the half–wave circuit.

The standard rectifier diode, being just a two terminal device, has no internal means of regulating current. It is often used, however, in conjunction with controllable rectifiers such as SCRs in circuits similar to the hybrid bridge rectifier shown in Figure 90.

Similar circuits have a multitude of applications but are most often used with fixed input voltages to vary the average power to the load. A change in gate pulse delay to the 5CR serves to vary the conduction angle, α, as desired. The hybrid bridge may also be used to regulate the load power over a range of input voltages or perform a combination of these functions. In such circuits the form factor can be quite high. For this reason, additional derating must be applied to the rectifier diodes current rating when operation is confined to small angles of conduction.

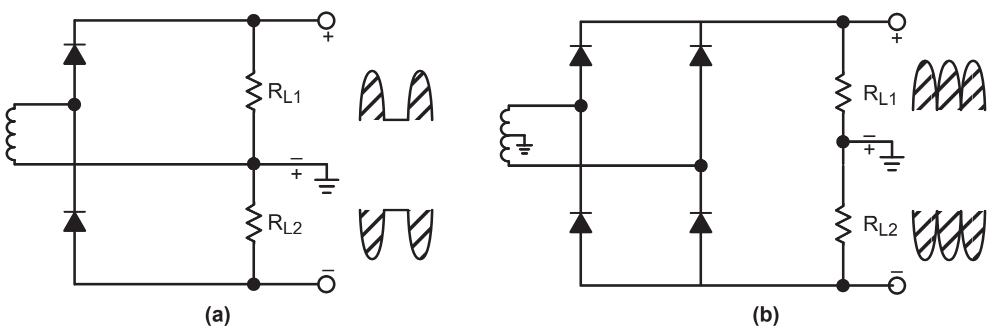


Figure 89. Single–Phase Supplies with Dual Output Voltages: (a) Dual Half–Wave Supply; (b) Dual Full–Wave Supply (a bridge assembly is convenient to use in this application)

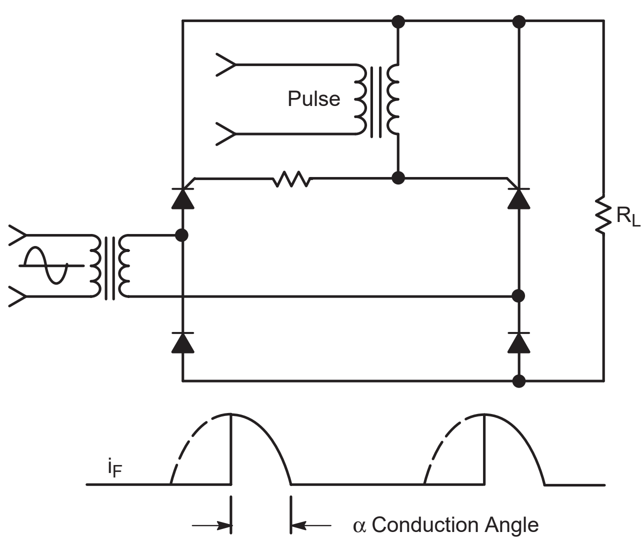


Figure 90. Typical Hybrid Bridge Rectifier with Resulting Diode Current Waveform

Summary

All the commonly used characteristics of low–frequency single–phase rectifier circuits with resistive loads have been discussed in this chapter. Results are summarized in Table 16. From the standpoint of maximum utilization of the transformer and reverse voltage rating of the rectifier diodes, the bridge circuit is superior to the other two. Although it requires more diodes, overall cost is generally similar because of simpler transformer construction and lower diode V_{RRM} requirements. At voltages of 12 V_{DC} and below, the power efficiency of the bridge becomes unacceptable because the output current must pass through two diodes in series. Furthermore, the diode V_{RRM} rating is of little concern. The full wave center–tap circuit is therefore preferable at low voltages. For economy reasons, the half–wave circuit finds use when current requirements are small, particularly if direct connection to the power line is allowable.

Filters are generally used on the output of single phase rectifying circuits. Circuit characteristics are similar to those shown in Table 16 when inductive loads or choke input filters are used, but characteristics change drastically with capacitive loads or filters. Chapter 7 discusses filtering and its effect upon rectifier circuit performance.

Chapter 6

Polyphase Rectifier Circuits

Polyphase Rectifier Circuits

A multiplicity of polyphase circuits have been devised, but only a few are of relative importance. This chapter develops some of the basic relationships to show the demands placed upon the rectifier diode by some of the more popularly used circuits. More exhaustive studies are given in the references [1] [2].

Polyphase transformer windings may be wye (Y) (also called "star") or delta (Δ) connected. When Y connected as in Figure 91(a), voltages from any terminal (i.e.. A, B, or C) to the neutral (N) are called phase voltages and appear on a vector diagram as shown in Figure 91(b). The voltages V_{AB}, V_{BC}, and V_{CA} are called line voltages. The line voltages are also 120° out of phase, but because they are the vector sum of two phase voltages they are $\sqrt{3}$ times larger and are displaced 30° from the phase voltages as shown in Figure 91(c). A common method of transformer connection is the delta connection indicated in Figure 91(d). The voltage across the delta terminal is the same as the line voltage, and consequently bears the same relationship to the phase voltage as the line voltage.

By summing voltages from two phases in a transformer, a voltage bearing any desired amplitude and phase shift from a phase voltage may be obtained. Thus, there is no inherent limit to the number of phases which may be generated.

When a three–phase system is driving a balanced load, the power is given by

$$P = \sqrt{3}\,VI, \tag{6.1}$$

where P = total power

V = rms phase or line voltage

I = corresponding rms phase or line current.

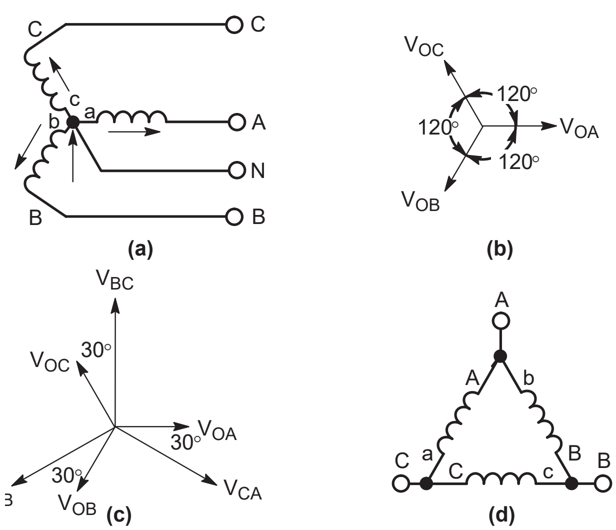


Figure 91. Y and Δ Connections and Appropriate Phase Relationships: (a) Wye–Connected Transformer Windings; (b) Phase Voltage Relationships; (c) Complete Voltage Vector; (d) Delta–Connected Transformer Windings

General Relationships in Polyphase Rectifiers

The analysis of polyphase rectifier circuits may be greatly simplified if the transformers and rectifiers are idealized; that is, they are assumed to possess no resistance or leakage reactance. Furthermore, the diode is assumed to have a zero forward voltage drop and a zero reverse current. By idealizing the components, simple general expressions can be derived for polyphase rectifiers.

Figure 92 shows the rectified voltage waveform for the general case of a polyphase rectifier circuit (See Figure 93 for the simplest polyphase rectifier circuit). Conduction takes place through the rectifier with the highest voltage across it. Note that neither the voltage nor the current is ever zero in the load. The instantaneous load voltage VL of Figure 92 is equal to the voltage of the conducting phase, and is given by

$$v_L = V_M \sin\theta \tag{6.2}$$

where V_M = the peak phase–to–neutral voltage.

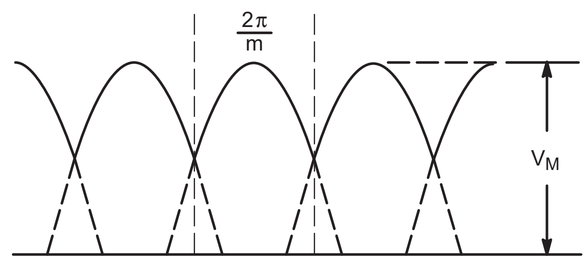


Figure 92. Rectified Voltage Waveforms for Multiphase Rectifier Circuit. The number of output pulses per cycle is designated

Current Relationships

As with single–phase circuits, the average or dc value of output current is of prime interest. It is the average value of the instantaneous rectified current over one cycle or a period of 2π/m radians (see Figure 92). Using integral calculus, the average current per phase equals the rectifier diode current and is given by

$$I_{F(AV)} = \frac{I_M}{2\pi}\int_{(\pi/2)+(\pi/m)}^{(\pi/2)+(\pi/m)} \sin\theta d\theta \tag{6.3}$$

Solving,

$$I_{F(AV)} = \frac{I_M}{\pi}\left[\sin\left(\frac{\pi}{m}\right)\right] \tag{6.4}$$

The total load current (I_L) is in times the current per conducting phase or

$$I_{L(DC)} = I_M\left(\frac{m}{\pi}\sin\frac{\pi}{m}\right) \tag{6.5}$$

The rms current (I_f) is also of interest as it determines heating in the transformer winding resistance and in the diode. The rms value varies as the square of the instantaneous current and may be obtained by integration.

$$I_f = I_{F(RMS)} = \left[\frac{1}{2\pi}\int_{(\pi/2)+(\pi/m)}^{(\pi/2)+(\pi/m)} I_M^2 \sin^2 \theta d\theta\right]^{1/2} \quad (6.6)$$

Solving:

$$I_f = I_{F(RMS)} = I_M\left[\frac{1}{2\pi}\left(\frac{\pi}{m} + \frac{1}{2}\sin\frac{2\pi}{m}\right)\right]^{1/2} \quad (6.7)$$

It is convenient to have the diode rms current in terms of average current rather than the peak value so that the form factor of rectifier current may be determined. Equation 6.4 may be combined with Equation 6.7 to obtain

$$F = \frac{I_{F(RMS)}}{I_{F(AV)}} = \frac{\sqrt{\frac{\pi}{2}\left(\frac{\pi}{m} + \frac{1}{2}\sin\frac{2\pi}{m}\right)}}{\sin\frac{\pi}{m}} \quad (6.8)$$

For m greater than or equal to three, Equation 6.8 may be approximated as

$$F = \sqrt{m} \quad (6.9)$$

The simplification is done by using the small angle approximation, $\sin\theta = \theta$. Ordinarily, the error caused by the approximation would be too large for $\theta = 60°$, but because of the nature of Equation 6.8, the overall error is small. For example, for m = 3 Equation 6.8 yields F= 1.76 while Equation 6.9 yields F= 1.73.

Voltage Relationships

In order to design the transformer, the required input voltage from each transformer secondary leg (V_i) for a given output voltage must be known. Since $V_M = R_L I_M$ and $V_{L(DC)} = R_L I_{L(DC)}$, substituting these relationships into Equation 6.5 results in

$$V_{L(DC)} = V_M\left(\frac{m}{\pi}\sin\frac{\pi}{m}\right) \quad (6.10)$$

The rms value of V_i is $V_M\sqrt{2}/2$. Substituted and rearranging Equation 6.10 yields

$$V_i = \frac{V_{L(DC)}}{\sqrt{2}\left(\frac{m}{\pi}\sin\frac{\pi}{m}\right)} \quad (6.11)$$

The peak repetitive reverse voltage or peak inverse voltage applied to the rectifier, V_{RRM}, is also of vital interest. In full–wave center–tapped circuits, it is the sum of the instantaneous load voltage and peak input voltage.

To obtain exact values, the precise output waveform must be considered. However, V_{RRM} cannot exceed 2 V_M and in many polyphase circuits $V_{L(DC)}$ approaches V_M. To be conservative, let $V_L = V_M$; since $V_{i(PK)} = V_L\sqrt{2}$, using Equation 6.11 above, V_{RRM} can be put in terms of voltage as

$$V_{RRM} \le 2\left(\frac{V_{L(DC)}}{\left(\frac{m}{\pi}\sin\frac{\pi}{m}\right)}\right) \quad (6.12)$$

For bridge connections, V_{RRM} is simply the peak output voltage, which is one half of that given by Equation 6.12.

Transformer Utilization

High transformer utilization is one of the chief benefits of polyphase rectifier systems. The utilization factor (UF) is the ratio of do power in the load to the volt ampere or power rating of the transformer and is commonly used in comparing rectifier circuits. As defined,

$$UF = \frac{I_{(AV)}V_{(AV)(OUTPUT)}}{I_{(RMS)}V_{(RMS)(INPUT)}} \quad (6.13)$$

Values for the terms depend upon whether primary or secondary UF is desired. The utilization factor of the primary is found by considering the total volt–ampere requirement of the transformer and the total do load power. The secondary utilization factor generally considers only a particular winding's contribution to the load and the volt–ampere requirement of the winding.

Utilization factors can only be computed by studying the voltage and current relationships in a particular circuit configuration. It can be shown [1] that the highest obtainable utilization factor occurs when in = 3, i.e., conduction occurs in a winding for 120°. In this case, the winding is idle for two–thirds of the cycle. The larger the number of phases, the longer is the idle time and the less effectively the transformer is being used. To achieve high utilization factors and also the advantages of 6– or 12–phase operation, bridge circuits are used and special secondary winding arrangements have been devised which permit m to be three for the secondary windings; however, in is effectively six or twelve for the output voltage waveform.

Ripple

The same relationships for ripple hold true for polyphase circuits as with single–phase circuits. In Chapter 5, it is shown that the ripple factor of the output voltage may be expressed as

$$r_f = \frac{V_{mn}}{\sqrt{2}V_{L(DC)}} = \frac{\sqrt{2}}{(nm)^2 - 1} \quad (6.14)$$

An expression for the various ripple harmonics is obtained for the general polyphase rectifier system by applying Fourier analysis to the waveform. The resulting expression is given by

$$r_f = \frac{V_{mn}}{\sqrt{2}V_{L(DC)}} = \frac{\sqrt{2}}{(nm)^2 - 1} \quad (6.15)$$

where

r_f = ripple factor due to the n^{th} harmonic,
V_{mn} = peak voltage of the n^{th} harmonic,
$V_{L(DC)}$ = dc output voltage
m = number of output pulses per cycle,
n = order of the harmonic.

In Equation 6.15, the only harmonics involved are multiples of in. Thus, for m = 3, only the third, sixth, ninth, etc., harmonics of the input voltage contribute to the ripple voltage.

Rectification Ratio

The last general item of interest is the rectification ratio Ó. sometimes referred to as waveform or conversion efficiency. It is the ratio of dc power in the load ($I^2_{L(DC)}R_L$) to rms power in the load ($I^2_{L(RMS)}R_L$). In the ratio of terms, the load resistance cancels and $I_{L(RMS)}/I_{L(DC)}$ is the load current form factor. Consequently, the rectification ratio may be expressed as

$$\sigma = \frac{1}{[F(load)]^2} \qquad (6.16)$$

To find the load current form factor, the total rms component of load current must be known. It may be found from Equation 6.6 by recognizing that the total current will be m times the phase current. Therefore.

$$I_{L(RMS)} = \left[\frac{m}{2\pi}\int_{(\pi/2)+(\pi/m)}^{(\pi/2)+(\pi/m)} I_M \sin^2\theta d\theta^2\right]^{1/2} \qquad (6.17)$$

Solving,

$$I_{L(RMS)} = I_M\left[\frac{m}{2}\left(\frac{\pi}{m} + \frac{1}{2}\sin\frac{2\pi}{m}\right)\right]^{1/2} \qquad (6.18)$$

By comparing Equation 6.1810 Equation 6.7, it can be seen that rms load current is simply √m times rectifier leg current and the load current is in times the rectifier leg current. Therefore, the form factor of the load is simply 1/√m times the rectifier leg form factor as given by Equation 6.8. Accordingly:

$$F(load) = \frac{\sqrt{\frac{\pi}{2m}\left[\frac{\pi}{m} + \frac{1}{2}\sin\frac{2\pi}{m}\right]}}{\sin \pi/m} \qquad (6.19)$$

An attempt to simplify Equation 6.19 as done with Equation 6.8 yields unity. Therefore, it must be used without approximation to yield meaningful results. Substituting Equation 6.19 into Equation 6.16 yields

$$\sigma = \frac{2m\sin^2(\pi/m)}{\left(\frac{\pi}{m} + \frac{1}{2}\sin\frac{2\pi}{m}\right)} \qquad (6.20)$$

Overlap

In practice current does not switch instantaneously from one diode to another Overlap is observed, that is, a period of time where current flows in two diodes, each in different branches. Overlap is caused because leakage inductance and the stored charge in diodes will not allow current in the individual phases to change instantaneously.

One effect of overlap is to reduce the output of the rectifier circuit. This reduction, along with the voltage drop in the rectifier and transformer should be calculated and used to correct the output voltage predicted ideally. The overlap problem is discussed more fully in Chapter 7 and in the references.

Common Polyphase Rectifier Circuits

A discussion of the basic and more popular rectifier circuit configurations follows. Pertinent characteristics of each circuit are listed in Table 18.

Three–Phase Half–Wave Star Rectifier Circuit

The three–phase star rectifier circuit, often referred to as the three–phase half–wave rectifier, is illustrated in Figure 93(a). The associated voltage waveforms are shown in Figure 93(b). Because the circuit is economical, it finds limited use where dc output voltage requirements are relatively low and current requirements are too large for practical single–phase systems. The circuit is worth studying mainly because it is a building block of more complicated systems. By using the equations developed in the preceding section, the various items of interest may be calculated as given in Table 18.

The dc output voltage is approximately equal to the phase voltage. However, the diodes must block approximately the line–to–line voltage, which is √3 times the phase voltage. In addition, the transformer design and utilization are somewhat complicated in order to avoid transformer core saturation caused by the do component of current flow in each secondary winding.

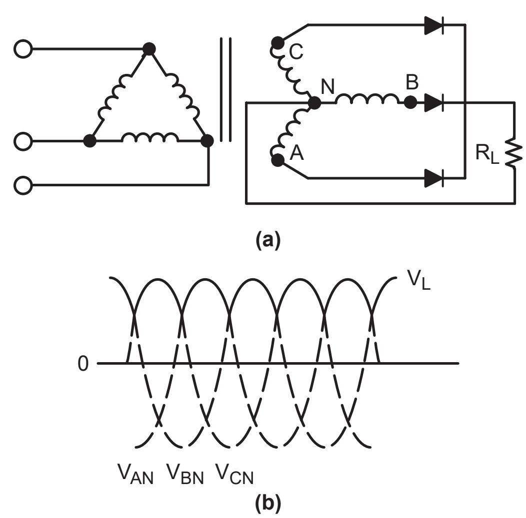


Figure 93. Circuit (a) and Waveform (b) for the Three–Phase Star or Half–Wave Rectifier Circuit

Three–Phase Inter–Star Rectifier Circuit

The three–phase inter–star or zig–zag rectifier circuit shown in Figure 94 overcomes some of the transformer limitations of the three–phase half–wave star circuit. The primary and secondary windings each consist of two coils, with pairs of coils forming a phase, located on different branches of the transformer core. The windings on the same core branch are connected such that the instantaneous magnetomotive force is zero. Although this connection eliminates the effects of core saturation and reduces the primary rating to the minimum of 1.05, it does so at the expense of economy, since it does not utilize the voltage of each winding to yield the highest possible out put voltage. The two secondary windings in series give a voltage of $\sqrt{3}V_{S1}$ instead of $2V_{S1}$. This results from the addition of two sinor voltages that are 600 apart. Consequently. the secondary volt ampere rating increases to 1.71 from the 1.48 value shown in Table 18. Otherwise, circuit constants are the same as the three–phase star circuit and therefore are not listed in Table 18.

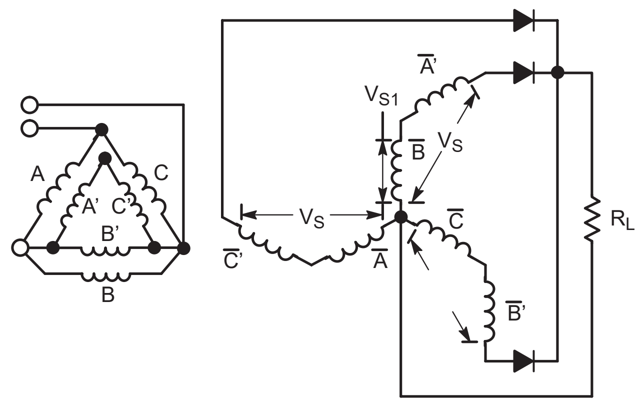


Figure 94. Three–Phase Inter–Star or Zig–Zag Circuit

Table 18. Summary of Significant Three–Phase Rectifier Circuit Characteristics for Resistive Loads

	Half–wave Star	Bridge	Double Wye with Interphase Transformer	Full–wave Star	Wye–Delta Connections	
					Parallel	Series
Average Current through Diode $I_{F(AV)}/I_{L(DC)}$	0.333	0.333	0.167	0.167	0.167	0.333
Peak Current through Diode $I_{FM}/I_{F(AV)}$	3.63	3.14	3.15	6.30	6.30	6.30
Form Factor of Current through Diode $I_{F(RMS)}/I_{(DC)}$	1.76	1.74	1.76	2.46	2.46	2.46
RMS Current through Diode $I_{F(RMS)}/I_{L(DC)}$	0.587	0.579	0.293	0.409	0.409	0.818
RMS Input Voltage Per Transformer Leg $V_i/V_{L(DC)}$	0.855	0.428	0.855	0.741	0.715	0.37
Diode Peak Inverse Voltage $V_{RRM}/V_{L(DC)}$	2.09	1.05	2.42	2.09	1.05	1.05
Transformer Primary Rating VA/P_{DC}	1.23	1.05	1.06	1.28	1.01	1.01
Transformer Secondary Rating VA/P_{DC}	1.50	1.05	1.49	1.81	1.05	1.05
Total RMS Ripple, %	18.2	4.2	4.2	4.2	1.0	1.0
Lowest Ripple Frequency, f_r/f_i	3	6	6	6	12	12
Rectification Ratio (Conversion Efficiency), %	96.8	99.8	99.8	99.8	100	100

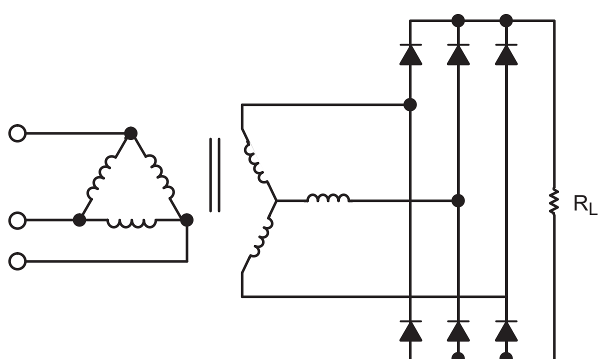
1. See Table 19 in Chapter 7 for inductive load data.

Three–Phase Full–Wave Bridge Circuit

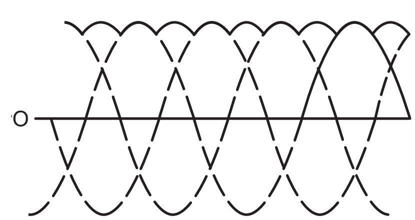
A three–phase full–wave bridge connection is commonly used whenever high dc power is required, as it exhibits a number of excellent attributes. It has a low ripple factor, low diode PIV, and the highest possible transformer utilization factor for a three–phase system. Because of the full–wave rectification associated with each secondary winding, it is permissible to use any combination of wye or delta primary and secondary windings or three single–phase transformers in place of one three–phase transformer. A schematic of a popular circuit is shown in Figure 95(a). The voltage waveforms are shown in Figure 95(b).

Each conduction path through the transformer and load passes through two rectifiers in series; a total of six rectifier elements are required. Commutation in the circuit takes place every 60°, or six limes per cycle. Such action is referred to as a six pulse rectifier which reduces the ripple to 4.2% and increases the fundamental frequency of ripple to six times the input frequency. No additional filtering is required in many applications. Thus, with this circuit the low ripple factor of a six–phase system is achieved while still obtaining the high utilization factor of a three–phase system. The dc output voltage is approximately equal to the peak line voltage or 2.4 times the rms phase voltage; each diode must block only the output voltage. Three–phase bridge connections are popular and are recommended wherever both dc voltage and current requirements are high.

The circuit characteristics are obtained by substituting m = 6 in the general equations. Results are shown in Table 18.



(a)



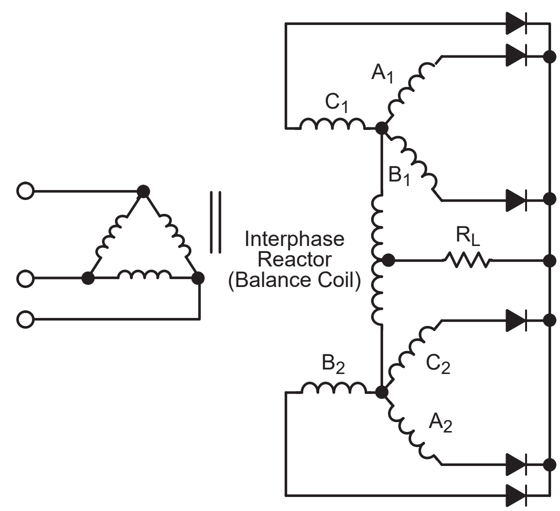
(b)

**Figure 95. Three–Phases Full Bridge Circuit and Associated Waveforms.
(a) Three–Phases Full–Wave Bridge Circuit;
(b) Voltage Waveforms, Solid Line is Output Voltage and the Dashed Lines are the Phase Voltages**

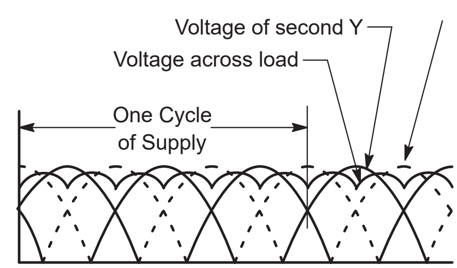
Three–Phase Double–Wye Rectifier with Interphase Transformer

The three–phase double–wye rectifier circuit is frequently used instead of a bridge circuit because each rectifier diode contributes only 1/6 instead of 1/3 of the load current. However, the peak inverse voltage for this circuit is higher than the three–phase star system due to the interphase reactor.

The circuit (Figure 96) consists essentially of two three–phase star circuits with their neutral points interconnected through an interphase transformer or reactor (also called a balance coil). The polarities of the corresponding secondary windings in the two parallel systems are reversed with respect to each other, so that the rectifier output voltage of one three–phase unit is at a minimum when the rectifier output voltage of the other unit is at a maximum, as shown. The action of the balance coil is to cause the actual voltage at the output terminals to be the average of the rectified voltages developed by the individual three–phase systems. The output voltage of the combination is therefore more nearly constant than that of a three–phase half–wave system; moreover, the ripple frequency of the output wave is now six times that of the supply frequency, instead of three times.



(a)



(b)

Figure 96. Circuit and Waveforms for the Three–Phase Double Wye Circuit: (a) Circuit; (b) Waveforms

In order that the individual three–phase half–wave systems may operate independently with current flowing through each diode one third of the time, the inter–phase reactor must have sufficient inductance so that the alternating current flowing in it as a result of the voltage existing across the coil has a peak value less than one–half the dc load current. That is, the peak alternating current in the interphase reactor must be less than the direct current flowing through one leg of the coil. Since the direct current flows in opposite directions in the two halves of the interphase reactor, no dc saturation is present in this reactor.

Six–Phase Star Rectifier Circuit

The six–phase star rectifier circuit shown in Figure 97 is often referred to as the three–phase diametric or full–wave rectifying circuit because it has a center–tapped transformer. The characteristics are obtained from the general rectifier equations where *m* is equal to six. Fields of application include requirements for very high dc load currents in low–to–medium voltage ranges. Voltage is usually restricted because the peak inverse voltage applied to the diodes is twice the peak phase voltage and transformer secondary utilization is poor. Current flows in only one rectifying element at a time, resulting in a low average current, but a high peak to average current ratio in the diodes.

The six–phase star circuit is attractive in applications which require a low ripple factor and a common cathode or anode connection for the rectifiers. The primary winding is generally delta–connected, although a wye connection is sometimes used with a tertiary winding. An additional advantage of the six–phase star is that the dc currents cancel in the secondary and, therefore, core saturation is not encountered.

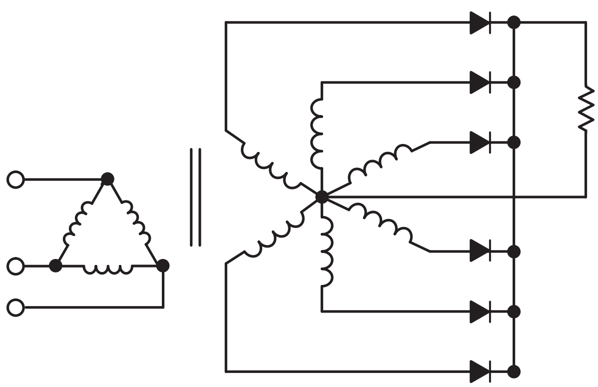


Figure 97. The Six–Phase Star Circuit

Six–Phase Full–Wave Bridge Circuits

In many applications, it is necessary to reduce ripple below the 4.2% level characteristics of tree–phase, full–wave and six–phase, half–wave connections. A reduction to approximately 1.0% at a ripple frequency twelve times the input frequency can be achieved by using wye–delta secondaries which result in 300 phase shift between windings. Either parallel or series bridge connections may be used for the output as shown in Figure 98.

When an equalizing reactor is used to couple the two bridge sections as in Figure 98(a), the system behaves as two parallel three–phase bridge circuits, with each section supplying one–half the load current. The parallel connection of the two groups is preferable to avoid having current pass through four diodes in series, but for high–voltage applications the series connection (as in part b) may result in a more economical design.

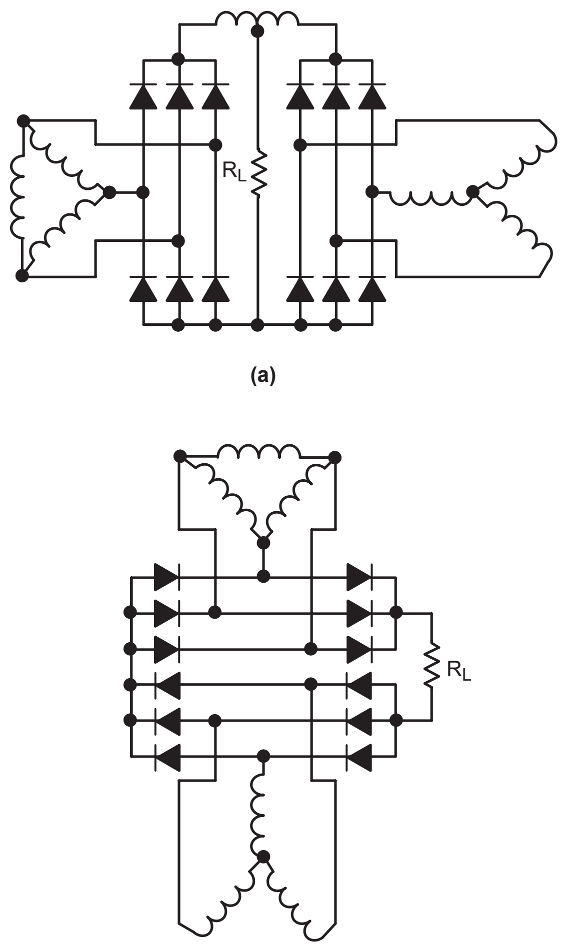


Figure 98. Six–Phase Full–Wave Bridge Circuits

Comparison of Circuits

As discussed, the three–phase and six–phase star circuits have limited applications because of a number of disadvantages. The most popular circuits by far are the three–phase full–wave bridge and the three–phase double–wye with interphase transformer. Both are alike in many respects.

Comparing the double–wye connection with interphase transformer with its rival, the three–phase bridge connection, it is found:

1. The diodes of one leg of an interphase transformer connection have to handle twice the voltage but only half the current of the diodes in one leg of a bridge connection with the same values of direct voltage and current. This is because the commutating groups operate in parallel in the interphase transformer connection and in series in the bridge connection. The high voltage rating is usually no problem and does not, in general, proportionally increase the price of the diodes (very high voltage excepted), but the lower current value reduces the rating of the diodes and all associated components such as fuses, bus bars, and cooling equipment, and is therefore a great advantage.
2. The total power losses are smaller in an interphase transformer connection because of the lower current carried by the diodes.
3. The reactive voltage drop caused by the bus bars can be made smaller in an inter–phase transformer connection since the current to be commutated is only half of the current commutated in a bridge connection, and the commutating voltage is twice as high.

For these reasons, the interphase transformer connection is competitive with the three–phase bridge connection in a certain voltage–current range, despite the relatively high transformer rating and the problem of avoiding saturation of the inter–phase transformer core due to current unbalance.

In special cases where low ripple is required and large power must be handled, a six–phase bridge or other high–order system may prove attractive. Other useful systems are covered in the literature [2].

High–Current Parallel–Connected Diodes

In some applications in chemical, metal, and transportation industries, the required dc output current cannot be achieved by using a single cell diode. In such instances, it is necessary to operate diodes in parallel. Care must be taken in the design to achieve a reasonable degree of current balance. The three main causes of unbalance are:

- differences in diode forward voltage drop
- self and mutual inductances of the diode and bus connection (referred to as the ladder effect in literature [3])
- magnetic coupling between legs.

Diode differences can be minimized by purchasing the diodes in closely matched sets but matched diodes are difficult to obtain from manufacturers at a cost–effective price. The resistance of the fuses used in series with each diode also improves balance. Some resistance will be encountered in the busing and connections. It should be very low in the main busses as compared to that in each individual diode leg.

Use of balancing transformers is a very effective means of forcing proper current balance and may be less expensive than purchasing factory matched units. The transformers consist of laminated iron cores usually with single–turn primary and secondary windings. The current from two diodes in parallel passes around the core in opposite directions so that any unbalance will induce a voltage which serves to correct the unbalance. The basic technique is shown in Figure 99. Its extension to larger numbers of rectifiers is illustrated and briefly discussed in Figure 100.

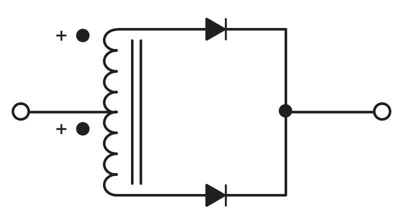
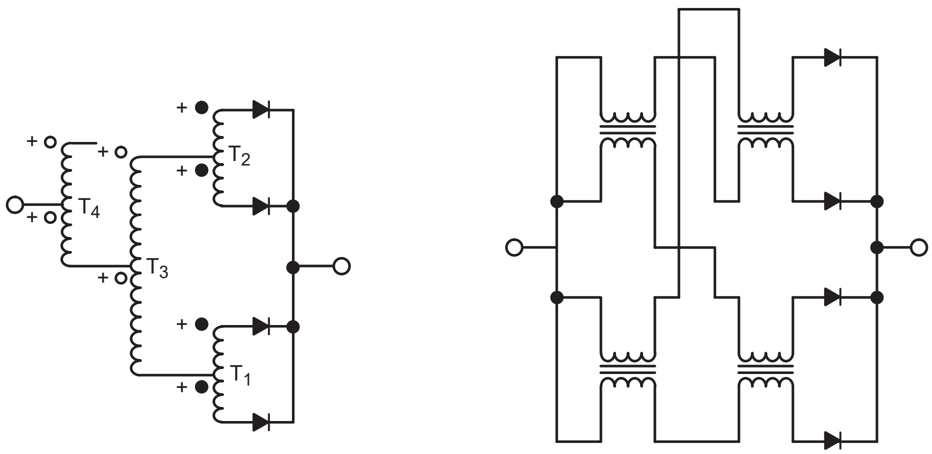


Figure 99. Parallel Operation of Rectifier Diodes Using Balancing Transformers

Computer simulation of the busing used in a stack of parallel diodes has led to the following design guidelines [4]:

1. The ac and dc buses of each leg should be as close as possible. Mutual inductance between these buses is beneficial.
2. The distance between the parallel diodes should be increased. The coupling of the mutual inductance of these paths acts as an improperly phased balance reactor.
3. The length of the diode path should be as short as possible. Although the resistance of the path increases as the path length increases, the self–inductance and mutual–inductance effects predominate, which increases the unbalance.



(a) (b)

Figure 100. Schemes for Balancing the Current When More than Two Diodes are Required.
(a) Balancing Reactors Used with Diodes in an Extension of the Basic Circuit. Transformer T_3 must handle twice the current of T1 or T2; T4 must handle twice the current of T3, etc.;
(b) Balancing Reactors in a Closed–Chain System. Each winding carries only the current of one diode

References

1. Jacob Millman and Samuel Seely, *Electronics,* Second Edition, Chapter 14, McGraw–Hill Book Company, Inc., New York, New York, 1951.
2. Johannes Schaefer, *Rectifier Circuits: Theory and Design,* John Wiley and Sons, Inc., New York, New York, *1965.*
3. D.A. Paice, *Multiple Paralleling of Power Diodes,* IEEE Trans. TECI, Vol. IECI22, pp. 151–158, May, 1975.
4. S.M. Peeran, A. Kusko, W.R. Hodgson & E.G. Fellendort, Current Balance in Parallel Power Diodes in Three–Phase Rectifiers, IEEE Transactions on Industry Applications, Vol. IA–23, No 3., May/June, 1987.

Chapter 7

Rectifiers Filter Systems

Rectifier Filter Systems

Rectifiers without output filters, especially single–phase circuits, find limited application owing to their high ripple and relatively low rectification ratio. A Fourier analysis of the rectifier output waveform yields a constant term (the dc voltage) and a series of harmonic terms. Filters are usually added to extract the constant term and attenuate all harmonic terms. The input impedance of the filter dramatically influences the current and voltage relations of the transformer–rectifier combination; therefore, filters are broadly classified by the type of input element used.

Inductor–input filters are preferred in higher–power applications in order to avoid excessive turn–on and repetitive surge currents. Choke–input filters also offer greatly reduced electromagnetic interference, which is frequently caused by rectifier repetitive surge currents. More efficient transformer operation is also obtained because of the reduction in form factor of the rectifier current.

An inductor attempts to hold the load current constant and is thus more effective at heavy loads or small values of load resistance. Use of an inductor alone is generally impractical, particularly when variable loads must be handled because the attenuation is not sufficient with reasonable values of inductance. Analysis of a single L–R network shows that the ripple is reduced by the factor:

$$\frac{1}{\sqrt{1 + (X_L/R_L)^2}} \qquad (7.1)$$

where

X_L is the reactance of the filter choke

R_L is the resistance of the load.

Unless $X_L \gg R_L$, filter action is ineffective. The basic inductor–input filter is therefore an L section employing both inductance and capacitance so that the complementary characteristics of the filter elements are used to advantage. It should also be noted that inductive loads, or filters, are not good practice in half–wave single–phase rectifiers owing to the large rectifier inverse–voltage transient which occurs when conduction inevitably ceases.

Demands for smaller and lighter equipment have forced designers to use capacitor–input filters in lower–power systems operating from single–phase ac lines. When capacitor–input filters are used, diodes whose average rating more nearly matches the load requirement can be used if a source–to–load resistance ratio of about 0.03 and voltage regulation of about 10% are acceptable. Close regulation from the rectifier is seldom needed in electronic equipment because the rectifier is followed by an electronic regulator or switch mode dc–to–dc converter.

Behavior of Rectifiers with Choke–Input Filters

The voltage and current relations existing in rectifier systems having a series–inductance (choke–input) filter are illustrated by the sketches of Figures 101 and 102. While these apply to specific rectifier circuits, the same general behavior occurs in all rectifier systems of the choke–input type.

When the input inductance is assumed infinite, the current through the inductance is constant and is carried at any moment by the rectifier diode which has the most positive voltage applied to its anode at that instant. As the alternating voltage being rectified passes through zero or when another anode becomes most positive, the current suddenly transfers from one diode to another, giving square current waves through the individual rectifier diodes, as shown by the dotted lines in the lower sketches of Figure 101. When the input inductance is finite and large, the situation is as shown by the solid lines. The current through the input choke increases when the output voltage of the rectifier exceeds the average or dc value and decreases when the rectifier output voltage is less than the dc value, causing the current through the individual diodes to be modified as shown. If the input inductance is too small, the current decreases to zero during a portion of the time between the peaks of the rectifier output voltage, and the conditions are similar to a capacitor–input filter system, as discussed later in this chapter.

The ratio of peak current per individual diode to the dc current developed by the overall rectifier system depends upon the rectifier connection and the size of the input inductance. When the input inductance is infinite, the anode of each rectifier diode must at some time during the cycle carry the entire load current, except in the case of the three–phase half–wave double–wye system, where each diode is called upon to carry only half of the load current. These and other useful current relations for different rectifier connections are presented in Table 19. When the input inductance is not infinite, the current through the input inductance will vary around the value for the infinite inductance case, as illustrated in Figure 101. In the least favorable case, corresponding to an input inductance barely sufficient to obtain choke–input type of operation, the peak value of diode current is twice the value corresponding to infinite input inductance.

Rectifier Applications

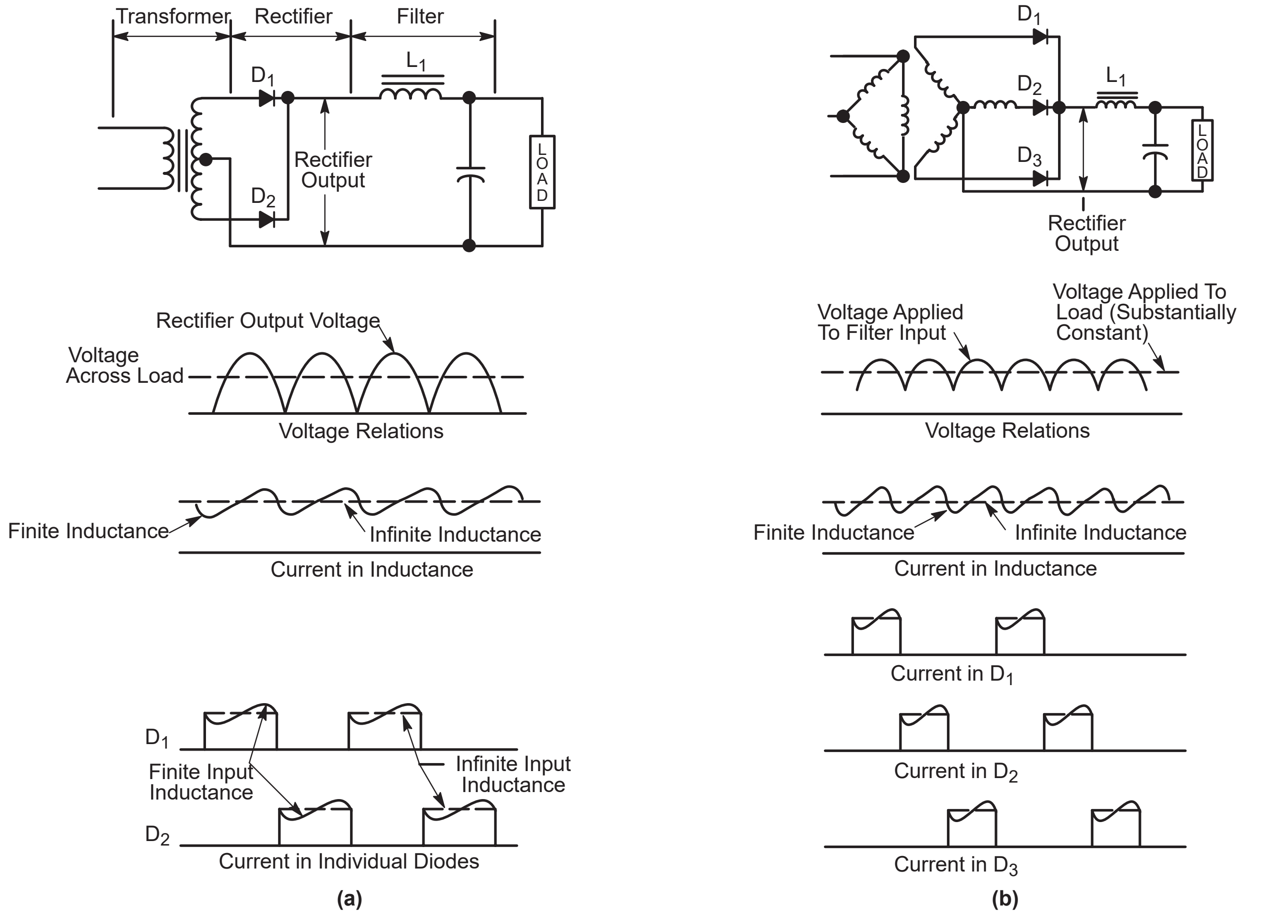


Figure 101. Voltage and Current Waveforms Existing in Rectifier Systems Operating with Choke–Input Filters: (a) Circuit of Rectifier and Filter, Single–Phase Full–Wave Case; (b) Circuit of Rectifier and Filter, Three–Phase Case

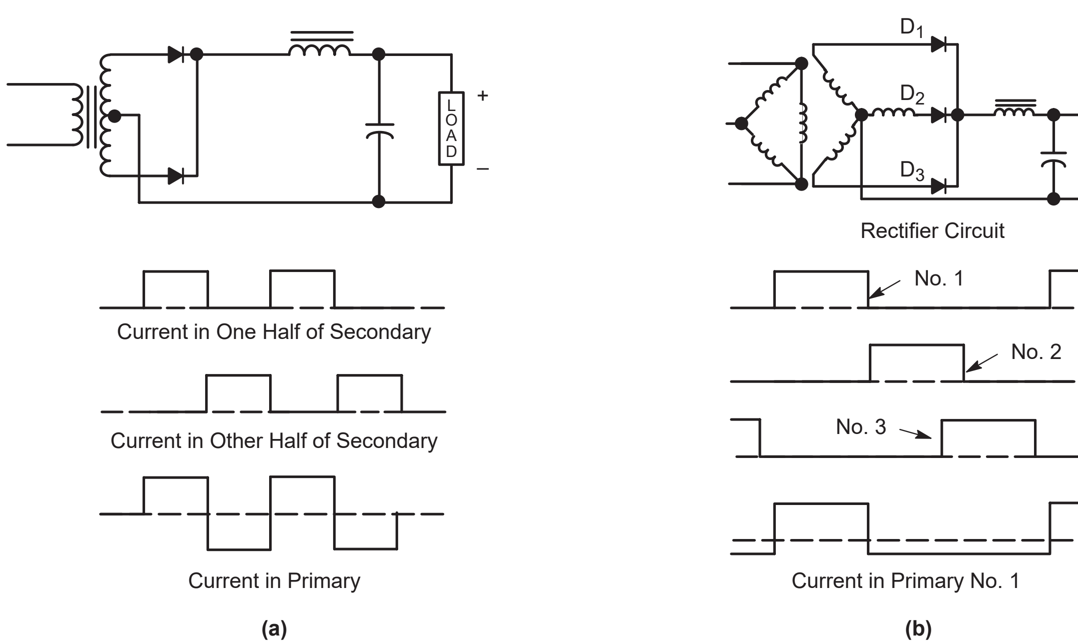


Figure 102. Typical Current Waves in Primary and Secondary Windings of Transformer for Ideal Choke–Input Systems: (a) Single–Phase Full–Wave System; (b) Three–Phase Half–Wave

Table 19. Characteristics of Typical Rectifiers Operated with Inductance–Input Filter Systems

Rectifier Circuit Connection / Characteristic	Single–Phase Full–Wave Center–Tap	Single–Phase Full–Wave Bridge	Three Phase Hall–Wave Star	Three–Phase Full–Wave Bridge	Three–Phase Double Wye With Interphase Transformer
‡Average Current Through Diode $I_{F(AV)}/I_{L(DC)}$	0.500	0.500	0.333	0.333	0.167
‡Peak Current Through Diode $I_{FM}/I_{F(AV)}$	2.00	2.00	3.00	3.00	3.00
Form Factor of Current Through Diode $I_{F(RMS)}/I_{F(AV)}$	1.41	1.41	1.73	1.73	1.76
RMS Input Voltage Per Transformer Leg $V_I/V_{L(DC)}$	1.11*	1.11	0.855	0.428	0.885
Diode Peak Inverse Voltage (PIV) V_{RRM} $V_{L(DC)}$	3.14	1.57	2.09	1.05	2.42
Transformer Primary Rating V_A/P_{DC}	1.11	1.11	1.21	1.05	1.05
Transformer Secondary Rating V_A/P_{DC}	1.57	1.11	1.48	1.05	1.48
Ripple ($V_F/V_{L(DC)}$ Lowest frequency in rectifier output (f_r/f_1) Peak Value of Ripple	2	2	3	6	6
Components: Ripple frequency (fundamental) Second harmonic Third harmonic	 0.667 0.133 0.057	 0.667 0.133 0.057	 0.250 0.057 0.025	 0.057 0.014 0.006	 0.057† 0.014 0.006
Ripple peaks with reference to dc axis: Positive peak Negative peak	0.363 0.837	0.363 0.637	0.209 0.395	0.0472 0.0930	0.0472 0.0930

1. This table assumes that the input inductance is sufficiently large to maintain the output current of the rectifier substantially constant, and neglects the effects of voltage drop in the rectifier and the transformers. $P_{DC} = 1^2{}_L R_L$, $V_L = I_L R_L$

*Secondary voltage on one side of center–tap.

†The principal component of voltage across the balance coil has a frequency of 3 fi and a peak amplitude of 0.500. The peak balance coil voltage, including the smaller, higher harmonics, is 0.605.

‡Assumes infinite input inductance.

The output voltage from the rectifier diodes applied to a choke input filter can, for nearly all practical purposes, be considered the same as if applied to a resistive load. The output voltage wave of the rectifier can be considered as consisting of a dc component upon which are superimposed ac voltages, termed ripple voltages. In the case of the idealized full–wave single–phase rectifier, Fourier analysis shows that the output wave v_L has the equation

$$V_L = \frac{2V_M}{\pi}\left(1 - \frac{2}{3}\cos\ 2\omega t - \frac{2}{15}\cos\ 4\omega t - \frac{2}{35}\cos\ 6\omega t\right) \quad (7.1)$$

where

V_M represents the peak value of the ac voltage applied to the rectifier diode

ω is the angular velocity ($2\ \pi f$) of the supply frequency.

(A full derivation of this equation is given at the end of this chapter.)

Note that the dc component of the output wave is $2/\pi$ times the crest value of the ac wave, and the lowest frequency component of ripple in the output is twice the supply frequency and has a magnitude that is two–thirds the dc component of the output voltage. The remaining ripple components are harmonics of this lowest frequency component and diminish in amplitude with the order of the harmonic involved in accordance with Equation 7.1. The more significant components of ripple are shown in Table 20. The ripple frequencies in the table are presented in relation to the fundamental frequency of ripple, not to the input frequency. For the full–wave circuit, the fundamental ripple component for a 60 Hz input is 120 Hz; the second harmonic is 240 Hz, and the third harmonic is 360 Hz.

Table 19 also gives the results of the analyses [1] for the output waves delivered by several popular polyphase rectifier connections. The ripple voltages are much less for the three–phase half–wave rectifier than for the single–phase connection, and are still less for the six–phase arrangements. In all cases, the amplitude of the ripple components diminishes rapidly as the order of the harmonics is increased. Not only are the ripple amplitudes lower but the ripple frequencies are higher for polyphase than for single–phase rectifiers, attributes which allow choke size to be greatly reduced. For example, the three–phase full–wave connections used with a 60 Hz input have a lowest ripple frequency component of 360 Hz with an amplitude of 5.7% of the dc level. The second harmonic is 720 Hz with a level of only 1.4% and the third harmonic is 1080 Hz with an amplitude of only 0.6%. Consequently, a relatively small choke and capacitor can reduce the fundamental ripple frequency to a very low level and the harmonics to an insignificant level.

Input Inductance Requirements

To achieve normal choke–input operation, it is necessary to maintain continuous flow of current through the input inductance. The peak value of the alternating current component flowing through the input inductance must, hence, be less than the dc output current of the rectifier. The value for minimum inductance, called critical inductance (L_C) is derived from the following argument.

The average or dc current is $V_{L(DC)}/R_L$. The peak alternating current is very nearly the peak value of the fundamental–frequency component, since the higher–frequency components of the ripple current are relatively small. The current ripple harmonics are smaller than those given by Table 19, because of the higher reactance of the inductor at the harmonic frequencies. The inductor is normally followed by a capacitor having a low reactance at the fundamental ripple frequency, so that the peak ripple component is very close to $V_{M1}/\omega_1 L$. The ratio of peak–to–average current therefore closely approximates $(V_{M1}/V_{DC})/\omega L_C/R_L$, which must not be less than unity.

For a single–phase, full–wave circuit, the fundamental ripple component ω_i is twice the input frequency ω_i; also, the ratio V_{M1}/V_{DC} is the coefficient (2/3) of the fundamental ripple frequency term (cos 2ωt) in Equation 7.1. After substituting $2\omega_i$, for ω_1 and 2/3 for V_{M1}/V_{DC} into the previous expression, the following equation for critical inductance is found:

$$L_C = \frac{R_L}{3\omega_i} \quad (7.2)$$

Using a similar procedure for polyphase rectifiers, the critical inductance is found to be

$$L = \frac{2R_L}{m(m^2 - 1)\omega_i} \quad (7.3)$$

In Equations 7.2 and 7.3

L_C = the critical inductance in Henries
R_L = the load resistance in ohms
ω_i = the input or line frequency in rad/sec
m = the effective phase number for the output wave.

(If the resistance of the choke, diode, or transformer is significant, the values should be added to R_L).

For convenience, Equations 7.2 and 7.3 are put in the form

$$L_C = \frac{R_L}{A} \quad (7.4)$$

where A is a constant determined from Figure 103.

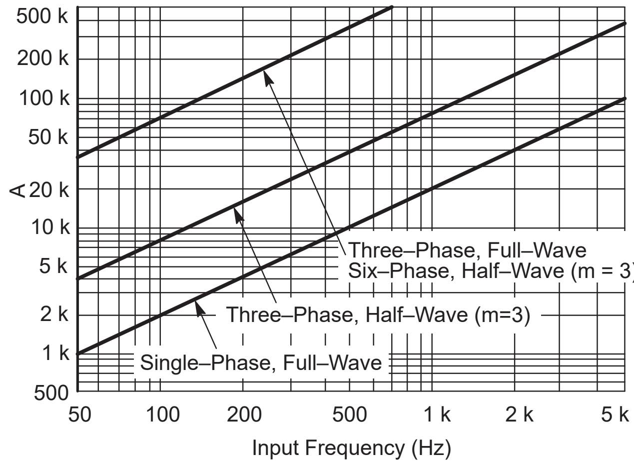


Figure 103. Values for the Constant A Used to Compute Critical Inductance

Values for other frequencies can easily be obtained by extrapolation, since A is inversely proportional to frequency. The value of A for a single–phase full–wave circuit operating at 20 kHz, for example, is 400,000 (10 times the 2 kHz value). The critical inductance required in polyphase systems is considerably less than that required in a single–phase system. The higher the load resistance (i.e., the lower the dc load current) the more difficult it is to maintain a continuous flow current. Also, with a given L, continuous flow will not occur when the load current drops below a critical value.

When the inductance is less than the critical value, the system acts similar to a capacitor–input system, described later. When the load current varies from time to time, it is necessary to satisfy the equations at all times if proper operation and, in particular, good voltage regulation are to be maintained. In order that this requirement may be satisfied at very small load currents without excessive inductance, it is usually necessary to place a resistance (commonly termed a bleeder resistance) across the output of the rectifier–filter system. The bleeder current is a compromise between having excessive dissipation in the bleeder resistor and excessively large L_c.

It is important to keep in mind that the effective inductance L_C is the incremental inductance, that is, the inductance to the alternating current superimposed upon the dc magnetization. Incremental inductance always increases as the dc magnetization decreases. This fact is of assistance in satisfying the equations at low load currents where R_L is large and can be put to advantage by the use of a “swinging” choke in which the inductance is varied by the load current using a controlled approach to saturation that lowers inductance as current increases.

It is essential to avoid series resonance between the input choke and first filter capacitor because at resonance very high circulating currents flow. Since at resonance, $1/\omega_1 L = \omega_1 C$, to avoid resonance it is sufficient to insure that

$$\omega_1^2 LC > 2 \qquad (7.5)$$

where ω is the lowest frequency component of the ripple.

The output ripple magnitude depends upon the values of the choke and the capacitor used. It is calculated as shown in the subsequent sections.

Voltage Regulation in Choke Input Systems

Given ideal components, the load regulation of a rectifier–filter system employing input inductance greater than the critical value would be perfect–i.e., voltage output would be independent of load current. In practice, the output voltage falls off with increasing load as a result of resistance in the diodes, filter, and transformer and as a result of the leakage reactance of the supply transformer. The various resistances in the circuit reduce the output voltage without affecting the waveshapes in the system. The leakage reactance of the transformer, however, distorts the waveshape of the output voltage by preventing the current from shifting instantly from one transformer winding to another, as in the ideal cases of Figure 101. The situation in a typical case is shown in Figure 104 where μ represents the time interval required to transfer the current. During this transition period, the output voltage assumes a value intermediate between the open–circuit voltages of the two windings that are simultaneously carrying current, instead of following the open–circuit potential of the more positive anode. As a result, the average voltage of the output is less than if no leakage inductance were present by the amount indicated by the shaded areas in Figure 104(c). The quantitative relations, which are quite complicated, depend upon both the rectifier and the transformer connections.

When the input inductance is less than the critical value, the output voltage rises, and when the load resistance is very large, the output voltage will approach the peak value of the rectifier output waveform causing the system to have poor voltage regulation. When the load current is small, the energy stored in the inductor is also small and the inductor is essentially out of the circuit.

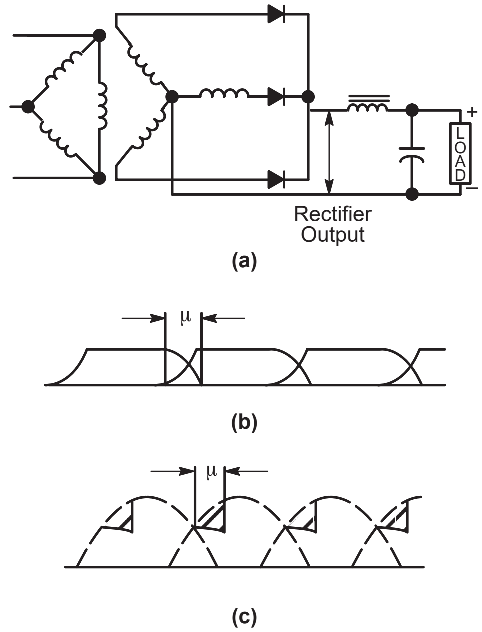


Figure 104. Effect of Transformer Leakage Inductance on the Behavior of a Polyphase Rectifier: (a) Three–Phase Half–Wave Rectifier Circuit; (b) Current of Individual Diodes; (c) Voltage Wave

Behavior of Rectifiers Used with Capacitor–Input Filters

The rectifier–filter system illustrated in Figure 105 differs from that of Figure 101 in that a shunt capacitor is presented to the rectifier output. Each time the positive peak alternating voltage is applied to one of the rectifier anodes, the input capacitor charges up to just slightly less than this peak voltage. No current is delivered to the filter until another anode approaches its peak positive potential. When the capacitor is not being charged, its voltage drops off nearly linearly with time because the load draws a substantially constant current. A typical set of voltage and current waveforms is shown in Figures 105(c) and 105(d). Use of an input capacitor increases the avenge voltage across the output terminals of the rectifier and reduces the amplitude of the ripple in the rectifier output voltage.

Analysis of Rectifiers with Capacitor–Input Systems

The detailed action that takes place in a capacitor–input system depends in a relatively complicated way upon the load resistance in the rectifier output, the input filter capacitance, the leakage reactance and resistance of the transformer, and the characteristics of the rectifier diode. For purposes of analysis, the actual rectifier circuit, such as that of Figure 105(a), is replaced by the equivalent circuit of Figure 105(b). The diode is replaced by switch S, which closes only when one of the diodes conducts current. The transformer is replaced by an equivalent generator having a voltage equal to the open–circuit line to center–tap secondary voltage and having equivalent internal impedance elements L_S and R_S. The inductance L_S is the leakage inductance of the transformer, measured across one–half the secondary winding with the primary short–circuited. The source resistance R_S the corresponding transformer's resistance plus a resistance to account for the resistance of the diode. In very low–voltage systems, a small potential ϕ may be needed to more accurately account for the diode voltage drop. The input capacitor of the filter system is C_1, and the first inductance L_1 (if used) and the load are assumed to draw a constant current I_1 equal to the dc voltage developed across the input capacitor divided by the sum of the actual load resistance plus the resistance of the filter inductance.

By utilizing the equivalent circuit of Figure 105(b), the effects that result from changes in individual circuit elements may be deduced. Thus, a decrease in the load resistance–i.e., an increase in the dc output current–reduces the average or dc output voltage, increases the ripple voltage, and increases the length of time during which the diode is conducting, as illustrated in Figure 106(a). Increasing the input capacitance has as its principal effect a decrease in the ripple voltage and also causes the average voltage to be increased slightly; these effects are shown in Figure 106(b). Increasing the leakage inductance (or resistance) of the transformer, reduces the average output voltage as illustrated in Figure 106(c), and likewise decreases the ratio of peak–to–average current flowing through the rectifier diode. In the case of a source and diode having very low impedance, the situation is as illustrated in Figure 106(d), and the peak current becomes quite high.

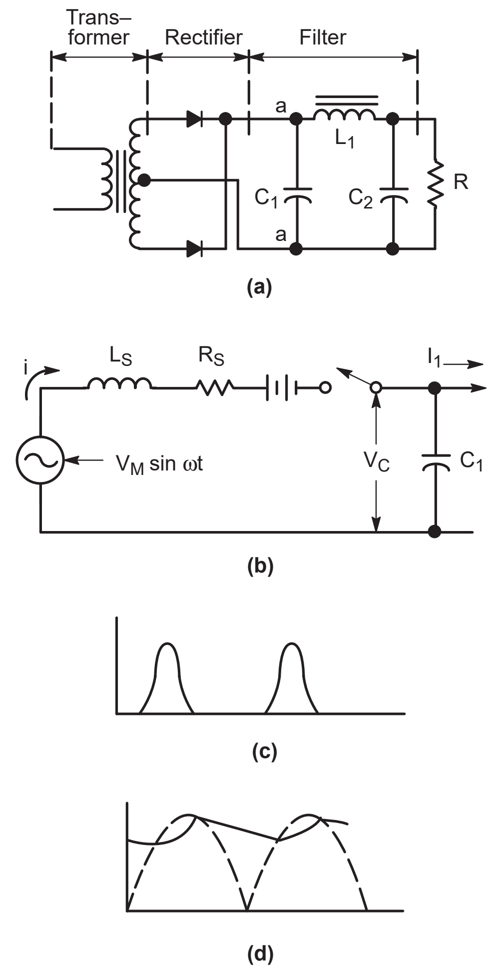


Figure 105. Actual and Equivalent Circuits of Capacitor–Input Rectifier System, Together with Oscillograms for Voltage and Current for a Typical Operating Condition:
(a) Actual Circuit; (b) Equivalent Circuit;
(c) Voltage Across Input Capacitor C1;
(d) Current Through Diode

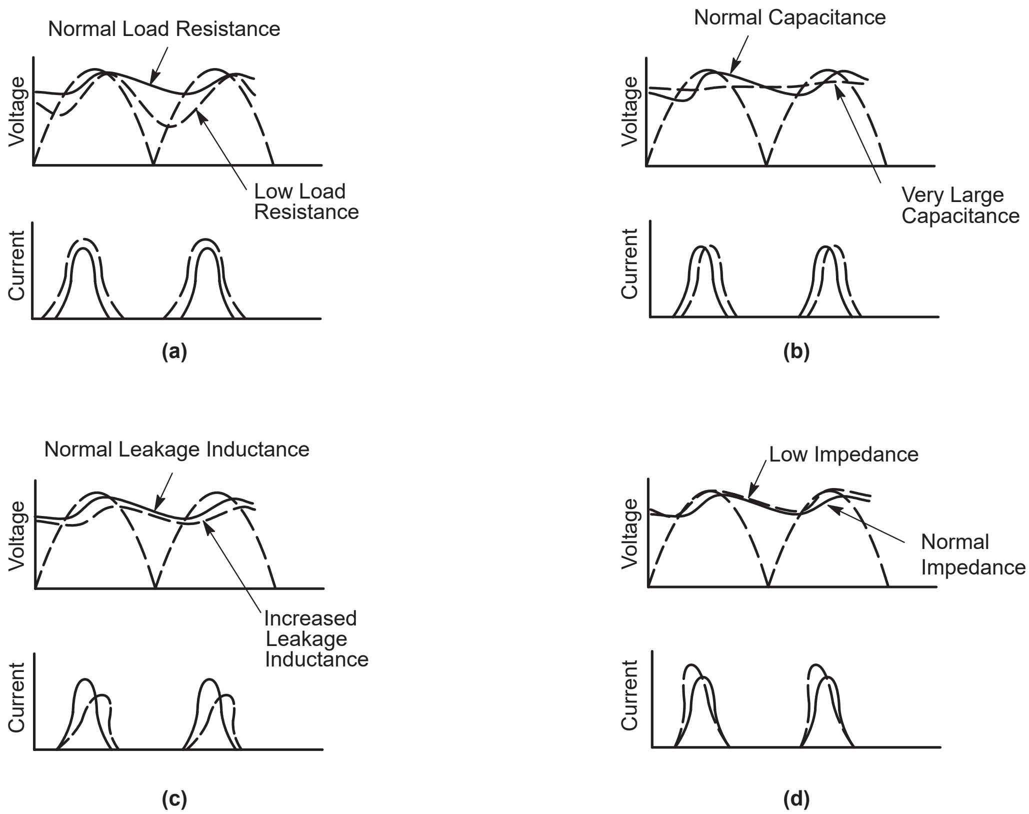


Figure 106. Effects of Circuit Constants and Operating Conditions on Behavior of Rectifier Having a Capacitor–Input Filter: (a) Effect of Load Resistance; (b) Effect of Capacitance; (c) Effect of Transformer Leakage Inductance; (d) Effect of Very Low Diode and Transformer Impedance

Design of Capacitor–Input Filters

The best practical procedure for the design of single–phase capacitor–input filters still remains based on the graphical data presented by Schade [2] in 1943. The curves shown in Figures 107 through 10 give all the required design information for half–wave and full–wave rectifier circuits. Whereas Schade originally also gave curves for the impedance of vacuum–tube rectifiers, the equivalent values for semiconductor diodes must be substituted. However, the rectifier forward voltage offset often assumes more significance than its dynamic resistance in low–voltage supply applications, as the dynamic resistance can generally be neglected when compared with the sum of the transformer secondary–winding resistance plus the reflected primary–winding resistance. The forward offset may be of considerable importance, however, since it is about 0.8 V for conventional silicon junction diodes and about 0.4 V for Schottky diodes, which clearly cannot be ignored in supplies for 12 V or less.

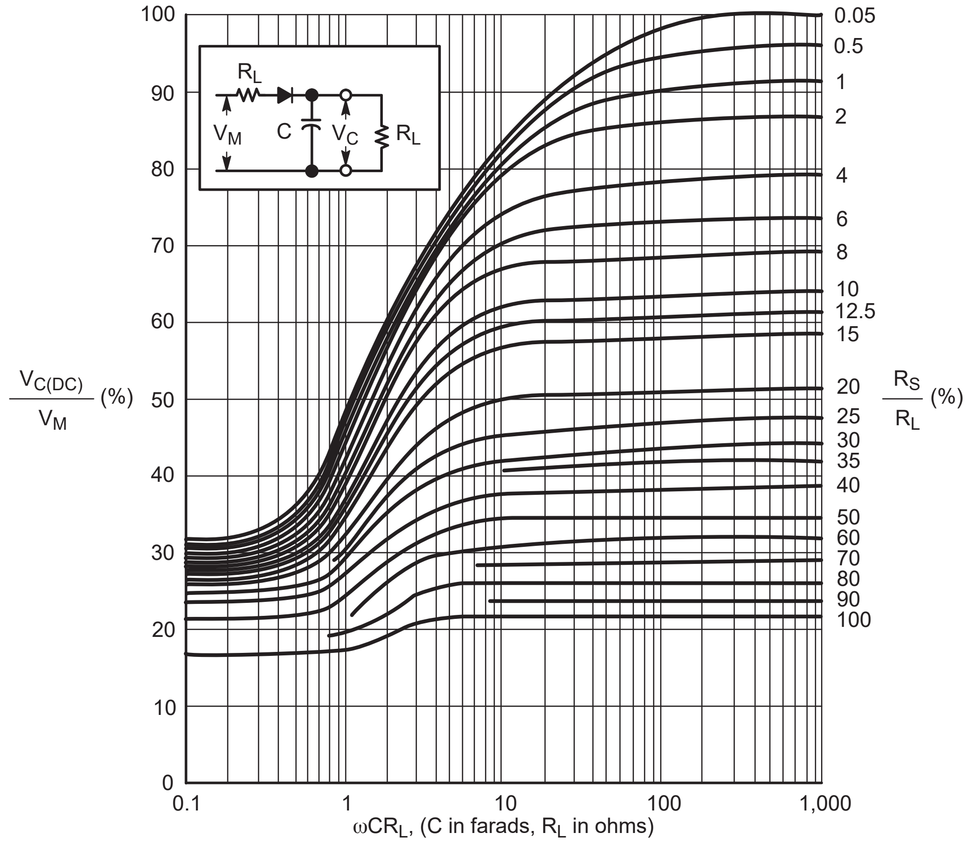


Figure 107. Relation of Applied Alternating Peak Voltage to Direct Output Voltage in Half–Wave Capacitor–Input Circuits (From O.H. Schade, Proc. IRE, Vol. 31, 1943, p. 356)

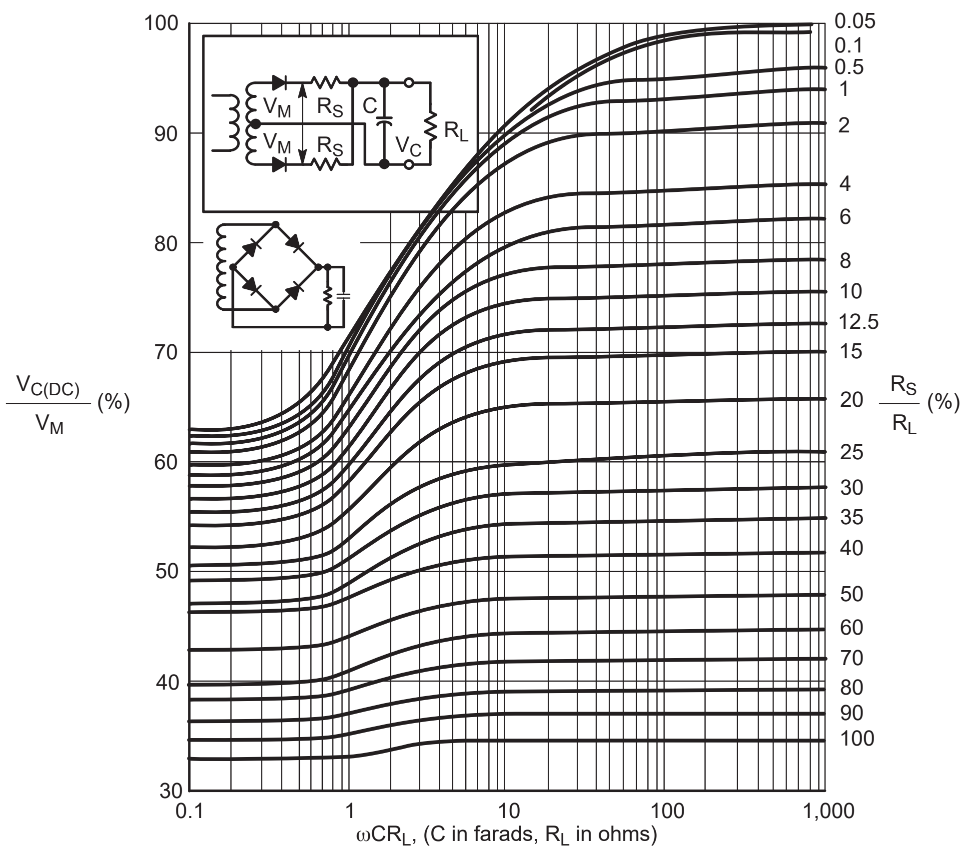
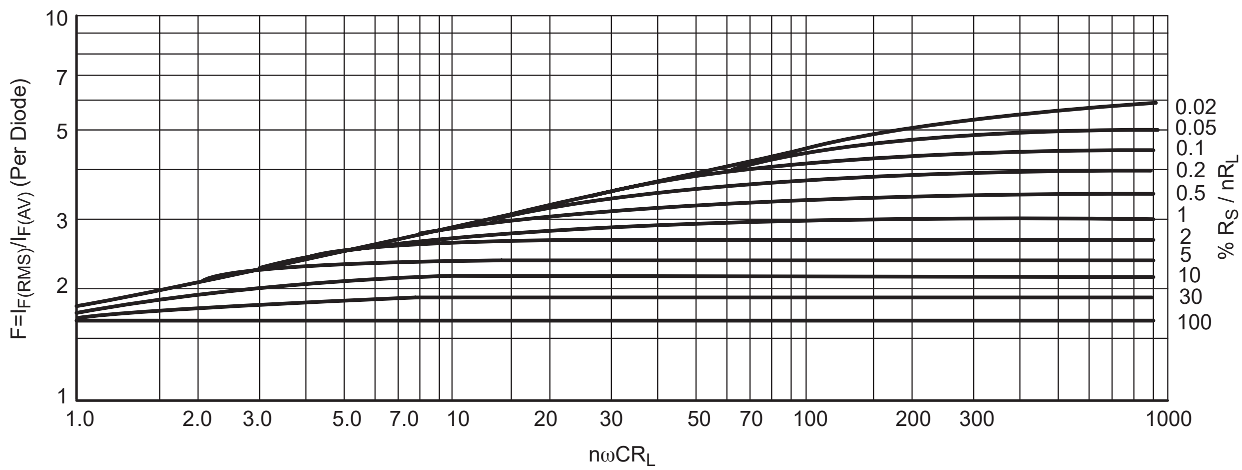


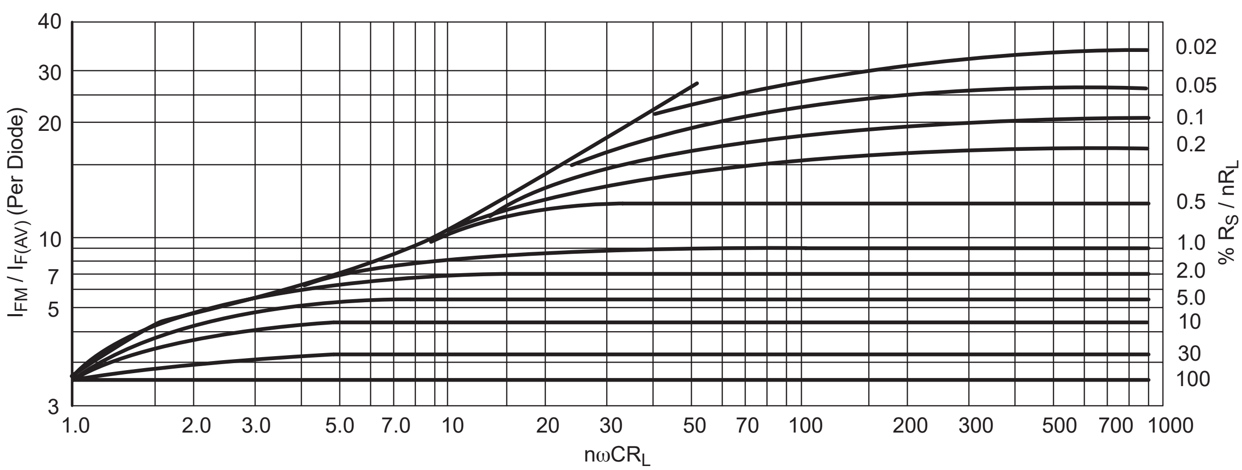
Figure 108. Relation of Applied Alternating Peak Voltage to Direct Output Voltage in Full–Wave Capacitor–Input Circuits (From O.H. Schade, Proc. IRE, Vol. 31, 1943, p. 356)

Figure 108 shows that a full–wave circuit must operate with $\omega CR_L \geq 10$ in order to hold the voltage reduction to less than 10 percent and $\omega CR_L \geq 40$ to obtain less than 2% reduction. However, these voltage–reduction figures require R_S/R_L, where R_S is now the total series resistance, to be about 0.1%. Such a low source resistance, if attainable, causes repetitive peak–to–average current ratios of 10 to 17 respectively, as can be seen from Figure 109. Rectifier diodes can handle high repetitive peak currents but their rated average current must be derated. To aid designers, manufacturers often provide capacitive load derating data.

The turn–on surge current generated when the input–filter capacitor is discharged and the transformer primary is energized at the peak of the input waveform causes a surge current determined by the peak secondary voltage less the rectifier forward drop limited only by the series resistance R_S. In order to control this turn–on surge, additional resistance must often be provided in series with each rectifier. It becomes evident that a compromise must be made between voltage reduction on the one hand, and diode surge rating and hence average current–carrying capacity on the other hand. If small voltage reduction–that is, good voltage regulation–is required, a much larger diode is necessary than that demanded by the average current rating. An obvious solution to this problem is to avoid the use of a capacitor–input filter. The alternative of a choke–input filter, as discussed previously, may be more costly than the cost of increased rectifier surge capacity.

A design example showing the use of Schade's curve is given at the end of this chapter. For comparative purposes, Table 21 shows rectifier circuit values when ωCR_L = 100 and R_S/R_L= 2%.





n = 1 For Half–Wave Single–Phase Rectifier Circuits
2 For Full–Wave Single–Phase Rectifier Circuits
ω = 2 πf, where f = Line Frequency

C in Farads
R_l in Ohms
R_S = RMD Equivalent Source Resistance

Figure 109. Relation of RMS and Peak to Average Diode Current in Capacitor–Input Circuits (From O.H. Schade, Proc. IRE, Vol. 31, 1943, p. 356)

Diode Peak Inverse Voltage

The peak inverse voltage applied to the diodes in a rectifier system operating in conjunction with a series–inductance input depends upon the topology. It ranges from as high as π times the dc component of the output voltage, in the case of the single–phase center–tapped connection, to barely more than the dc voltage in the three–phase full–wave bridge. Results in typical cases are given in Tables 19 and 21 and are the same as those determined for the resistive load circuits in Chapters 5 and 6.

The peak inverse voltage applied to the rectifiers in capacitor–input systems is generally based upon the capacitor charging to V_M which is the case when the load is light and the capacitor large. However, the peak of the reverse voltage sine wave occurs sometime after the capacitor has been charged. The peak voltage the capacitor attains is a function of the R_S/R_L ratio; the voltage on the capacitor when a diode's reverse voltage reaches its peak is dependent upon the load time constant. The relationship has been worked out for two cases as shown in Figure 111.

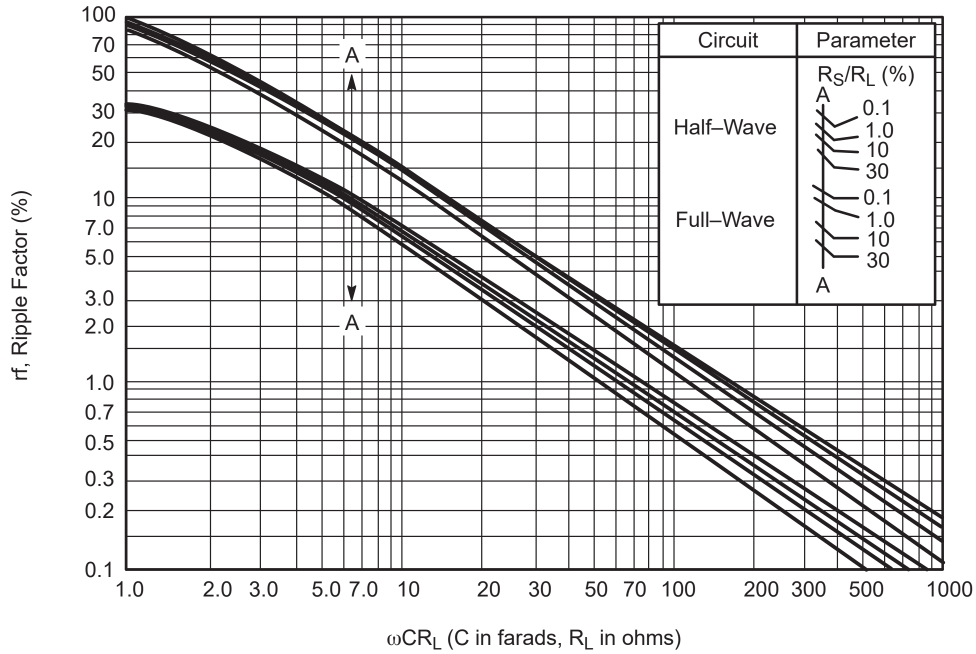


Figure 110. Root–Mean–Square Ripple Voltage for Capacitor–Input Circuits (From O.H. Schade, Proc. IRE, Vol. 31, 1943, p. 356)

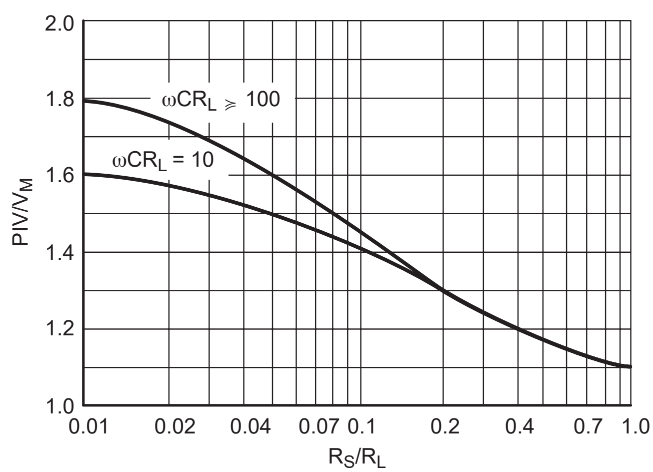


Figure 111. Ratio of Operating Peak Inverse Voltage to Peak Applied 60 Hz AC for Rectifiers Used in Capacitor–Input, Single–Phase, Filter Circuits

Transformer Considerations

The ratio of the actual dc rectified power output to the volt–ampere capacity of the windings on the basis of sinusoidal waves, for the same heat loss due to winding resistance, is termed the transformer utilization factor (UF). Its value depends upon the rectifier connections and is, in general, not the same for the primary and secondary windings, since the waveshapes in these windings will generally be different. Table 19 shows the reciprocal of the UF of the primary and secondary windings for some of the more commonly used rectifier connections. Derivation of the values is handled as shown for the resistive load circuits in Chapters 5 and 6.

The wave shapes of the currents that flow through the windings of the transformer in the idealized case of a rectifier system operating with an infinite input inductance are shown in Figure 102 for two typical cases. Because these waves are not sinusoidal, the heating of the transformer windings is greater for a given dc power output from the rectifier than would be the case if the same amount of ac power were delivered by the transformer to a resistance load. It is accordingly necessary to design the transformer windings more generously for rectifier applications than for cases where sinusoidal currents are involved.

Since transformer volt–ampere ratings vary with the rms content of the rectified circuit, they must be calculated for each capacitive–input filter design. Higher R_S/R_L ratios help reduce transformer requirements. Also, the effects of transformer saturation and finite bandwidth cannot be ignored. The former tends to limit the peak current while the latter allows the capacitor to charge over a longer time and hence reduces the peak charging current $i = C\ dv/dt$. Core saturation generally occurs on the recurrent charging peaks, and this will cause a lower output voltage than that calculated from the rated transformer secondary voltage. That saturation is occurring and that flattening is not due to the secondary–winding resistance can be determined by observing the voltage waveform induced in an unloaded winding on the same core.

Harmonics

Because the currents which flow in a rectifier circuit using a choke or capacitor input filter are not sinusoidal, high values of harmonic currents are present. These harmonic currents complicate the design of the transformer and, in single–phase circuits, the harmonic currents flow into the power line where they can cause a number of difficulties with the power distribution system. In a polyphase rectifier, the third harmonic can be kept from the powerline by having the primary windings arranged in a delta configuration. The circulating third harmonic component of primary current, of course, causes additional power loss in the transformer.

The choke input filter causes a square wave of current to flow in a single phase circuit. A Fourier analysis indicates that only odd harmonics are present and they decrease as 1/n, where n is the harmonic number. Thus, the amplitude of the third harmonic is 1/3, the fifth is 1/5, etc., that of the fundamental frequency.

Capacitive input filters cause a much higher current harmonic content than choke input filters cause. The harmonic content is particularly severe in transformer–less single phase supplies because transformer leakage inductance is not present. Inductance in series in the rectifier widens the diode conduction angle and lowers the current peak. A computer simulation of a single–phase bridge rectifier with a capacitor input filter yielded the results shown in Table 20 [3]. The value of the filter capacitor is normally so large that small changes in its value do not alter the results.

Table 20. Harmonic Currents In a Single–phase Full–wave Capacitor–Input Rectifier

Harmonic	Z_R*			
	1%	2%	5%	10%
3rd	83.0%	76.3%	63.5%	49.8%
5th	56.2%	42.4%	22.0%	10.2%
7th	29.2%	15.4%	8.1%	7.3%
9th	11.7%	8.7%	6.1%	3.8%
11th	8.6%	7.1%	3.5%	2.5%

*Z_R = Ratio of source inductive reactance to load impedance at the fundamental frequency.

The bridge is used without a transformer when its output is used by a switch mode dc–dc converter. In this case, the harmonic problems are transferred to the transformer in the electrical power distribution system feeding the equipment.

For a single–phase circuit, it is possible to significantly reduce harmonics fed into the power line by adding an optimally chosen inductor in series with the line to widen the conduction angle of diode forward current [4]. The inductor is much smaller than would be required for the typical continuous conduction choke–input filter. An optimally chosen capacitor added across the line raises the power factor to approximately 0.9, which is the same as obtained with a typical choke input filter.

Polyphase circuits behave differently. The larger the choke, the lower the harmonics and the higher the power factor. Increasing the inductor above a threshold does not offer any significant improvement in harmonics or power factor.

Surge Current

Because semiconductor junctions are limited in the amount of transient power that they can safely handle, attention must be given to the surge current occurring when the circuit is energized. The expected behavior can be deduced by studying the equivalent circuit of the rectifier system shown in Figure 105(b).

For analyzing a choke input filter, a choke is inserted between the switch and the capacitor C_1. Before voltage is applied, C_1 may be regarded as a short. Current buildup will be governed by the L/R_S time constant of the circuit. (L is the sum of the filter and transformer leakage inductance). The current surge is not significantly different than encountered in normal operation, even if the input voltage is applied at a time when the voltage has reached its positive peaks, because the inductor has high reactance to fast rising transients.

The capacitor–input filter, however, allows a large surge to develop, because the reactance of the transformer leakage inductance is rather small. The maximum instantaneous surge current is approximately V_M/R_S and the capacitor charges with a time constant $\tau = R_S C_1$. As a rough–but conservative–check, the surge will not damage the diode if V_M/R_S is less than the diode I_{FSM} rating and τ is less than the time of one–half cycle. It is wise to make R_S as large as possible and not pursue tight voltage regulation; therefore, not only will the surge be reduced but rectifier and transformer ratings will more nearly approach the dc power requirements of the rectifier load.

Comparison of Capacitor–Input and Inductance–Input Systems

The basis for distinguishing between inductance–input and capacitor–input systems is that in the former the current flows continuously from the rectifier output into the filter systems, while in the latter the current flows intermittently from the rectifier into the filter. Intermittent action is also present with inductance–input systems when the input inductance is less than the critical value. When this is the case, the system is classified as a capacitor–input arrangement even though it possesses a series inductance.

A comparison of the performance of inductance–input and capacitor–input systems shows that, in the latter arrangement, the dc voltage is higher, the ripple voltage more, the surge current higher, and the voltage regulation poorer than when inductance input is used with the same diode, transformer, load resistance, and capacitance. Also, with capacitor–input the ripple voltage increases with increasing load current, unlike the inductance–input system where the ripple voltage is independent of load current. The utilization factor of the power transformer is much poorer with the capacitor–input system because of the higher ratio of peak–to–average current flowing through the rectifier diode, and likewise the diode is less efficiently utilized. The characteristics of several popular rectifier systems are shown in Table 21.

Shunt capacitor–input arrangements are generally employed in most consumer equipment such as personal computers, radio and television receivers, high–fidelity sound systems, or small public–address systems, when the amount of dc power required is small. They must be used with half–wave rectifiers and are attractive when the input power is taken directly from the ac line without a power transformer. In contrast, inductance–input arrangements are used when the amount of power required is large, since then the higher utilization factor and lower peak currents result in important savings in rectifier and transformer costs. Inductance input is often employed when good regulation of the dc voltage is important but the precision of an electronic regulator circuit is not needed. Inductance–input systems are normal in polyphase rectifier systems.

Filter Sections

Figure 114 gives typical examples of filters that are placed between the rectifier output and the load to make the current delivered to the load substantially pure direct current. These filters are made up of series impedances (either inductances or. resistances) that oppose the flow of alternating current from the rectifier output to the load and shunt capacitors that bypass the alternating currents that succeed in flowing through the series impedances.

For purposes of discussion and analysis, filters are ordinarily divided into sections, each consisting of a series impedance followed by a shunt capacitor, as indicated in Figure 114. In this classification, the inductance in an inductance–input system is considered to be part of the first section of the filter. However, a shunt capacitance across the rectifier output is not included as part of a filter section, but rather is considered to be part of the rectifier system, which delivers to the first filter section a voltage corresponding to the voltage developed by the rectifier across the input capacitor.

Table 21. Summary of Significant Rectifier Circuit Characteristics. Capacitive Data is for WCRL= 100 and RS/RL = 2.0%

Rectifier Circuit Connection — Characteristic	Load	Single–Phase Half–Wave	Single–Phase Full–Wave Center–Tap	Single–Phase Full–Wave Bridge	Three–Phase Half–Wave Star	Three–Phase Bridge	Three–Phase Double–Wye With Interphase	Three–Phase Full–Wave Star
Average Current Through diode $I_{F(AV)}/I_{L(DC)}$	RL&C*	1.00	0.50	0.50	0.333	0.333	0.167	0.167
Peak Current	R	3.14	3.14	3.14	3.63	3.15	3.15	6.30
Through Diode	L*	–	2.00	2.00	3.00	3.00	3.00	6.00
$I_{FM}/I_{F(AV)}$	C	8.0	8.0	8.0	DATA NOT AVAILABLE			
Form Factor of	R	1.57	1.57	1.57	1.76	1.74	1.76	2.46
Current Through	L*	–	1.41	1.41	1.73	1.73	1.73	2.45
Diode $I_{F(RMS)}/I_{F(AV)}$	C	2.7	2.7	2.7	DATA NOT AVAILABLE			
RMS Current	R	1.57	0.785	0.785	0.587	0.579	0.293	0.409
Through Diode	L*	–	0.707	0.707	0.578	0.578	0.289	0.408
$I_{F(RMS)}/I_{L(DC)}$	C	2.7	1.35	1.35	DATA NOT AVAILABLE			
RMS Input Voltage	R&L	2.22	1.11	1.11	0.855	0.428	0.855	0.741
per Transformer Leg $V_i/V_{L(DC)}$	C	0.707	0.707	0.707	0.707	0.408	0.707	0.707
Diode Peak Inverse	R&L	3.14	3.14	1.57	2.09	1.05	2.42	2.09
Voltage PIV $V_{RRM}/V_{L(DC)}$	C	2.00	2.00	1.00	2.00	1.00	2.00	2.00
Transformer Primary	R	3.49	1.23	1.23	1.23	1.05	1.06	1.28
Rating V_A/P_{DC}	L	–	1.11	1.11	1.21	1.05	1.05	1.28
Transformer Secondary	R	3.49	1.75	1.23	1.50	1.05	1.49	1.81
Rating V_A/P_{DC}	L	–	1.57	1.11	1.48	1.05	1.48	1.81
Total RMS Ripple %	R	121	48.2	48.2	18.2	4.2	4.2	4.2
Lowest Ripple Frequency f_r/f_s		1	2	2	3	6	6	6
Rectification Ratio	R	40.6	81.2	81.2	96.8	99.8	99.8	99.8
(Conversion Efficiency) %	L*		100	100	100	100	100	100

*Inductive data valid when the circuit input voltage is a square wave, resistive or inductive load. $P_{DC} = I^2_L R_L$ (R_S neglected) $V_L = I_L R_L$

Voltage and Current Relations in Filters

The input voltage to the filter is whatever output voltage is developed by the rectifier connection in the case of inductance–input systems (it is the same as with a resistive load), or is the voltage developed across the shunt capacitor in the case of capacitor–input systems. The ripple voltage in the output of the rectifier–filter system is then the alternating component of this input voltage reduced by the action of the filter sections.

Most filter sections are composed of a series inductance and a shunt capacitance. In practical filters of this type, the reactance of the shunt capacitance at the lowest ripple frequency is much smaller than the resistance of the load (or the reactance of the series inductance of the following section). Substantially all ripple current entering the inductance L of the section flows through the capacitance C of the filter section. The current in the section is then $V_i/(\omega L - 1/\omega C)$, where V_i is the alternating voltage applied across the input to the filter section. The voltage that this current develops in flowing through the capacitor C is $(1/\omega C)V_i/\omega L - 1/\omega C) = V_i/\omega^2 LC - 1)$. Dividing by V_i results in the following relationship for the voltage reduction through an L–C section:

$$\frac{V_o}{V_i} = \frac{1}{\omega^2 LC - 1)} \qquad (7.6)$$

where

L = series inductance of filter section

C = shunt capacitance of section

ω = angular frequency of ripple voltage involved.

Results of Equation 7.6 are given in Figure 113 for the usual case where the ripple components are harmonics of

60 Hz. Figure 115 indicates the actual ripple output of a full–wave single–phase rectifier circuit.

When the current drawn by the load impedance is small, the series inductance of the LC filter may be replaced by a series resistance as shown in Figures 114(e) and 114(f). This arrangement is widely used in low–current applications such as resistance–coupled amplifiers, tuned radio–frequency amplifiers, and so forth, and has the advantage that a resistance is much less expensive than an inductance of corresponding effectiveness. The disadvantage is the dc voltage drop and power loss that occur in the resistance; these losses limit the resistance–capacitance type of filter to cases where the current is small and where a moderate dc voltage drop is permissible.

In practical resistance capacitance filters, the reactance of the shunting capacitor is always made small compared to the series resistance of the filter and to the impedance to which the output of the filter section is connected. The ripple current flowing in R is $V_i \sqrt{R^2 + (1/\omega C)^2} \approx V_i/R$, where V_i is the ripple voltage applied to the section. The ripple output voltage of the section results from the current V_i/R flowing through the reactance $1/\omega C$ of the capacitance C of the section, and hence is $V_i/R\omega C$. Therefore, a section of resistance–capacitance filter reduces each frequency component of voltage applied to the input side according to the approximate relation:

$$\frac{V_o}{V_i} = \frac{1}{R\omega C} \qquad (7.7)$$

where

R is the series resistance

C is the shunt capacitance.

(See Figure 114(e)).

When a large amount of ripple reduction is required, more than one section is used because the total capacitance and inductance is less than that required for just one section. The total reduction in ripple voltage produced by the several sections is very nearly the product of the voltage–reduction factors of the individual sections. The effectiveness of the filtering accordingly increases rapidly with the number of sections. For many applications, a single section is entirely adequate, particularly with polyphase rectifiers. Only in special cases, such as audio–frequency amplifiers with very high gain, will the number of sections required exceed two, and then only for those parts of the amplifier that operate at very low signal–power levels.

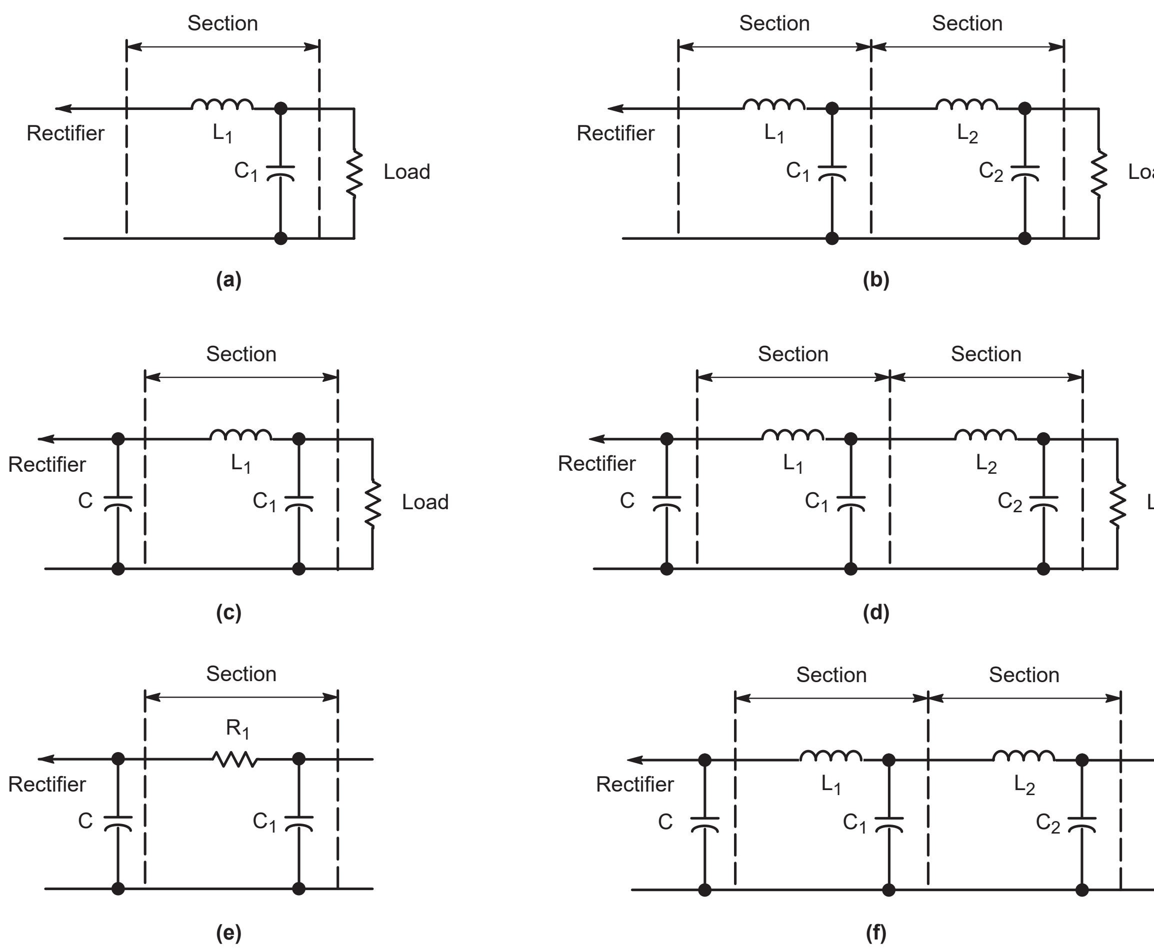


Figure 112. Typical L and π Section Filters: (a) and (b) L–C Filters, Choke Input; (c) and (d) L–C Filters, Capacitor Input; (e) and (f) Filters with Series Resistances

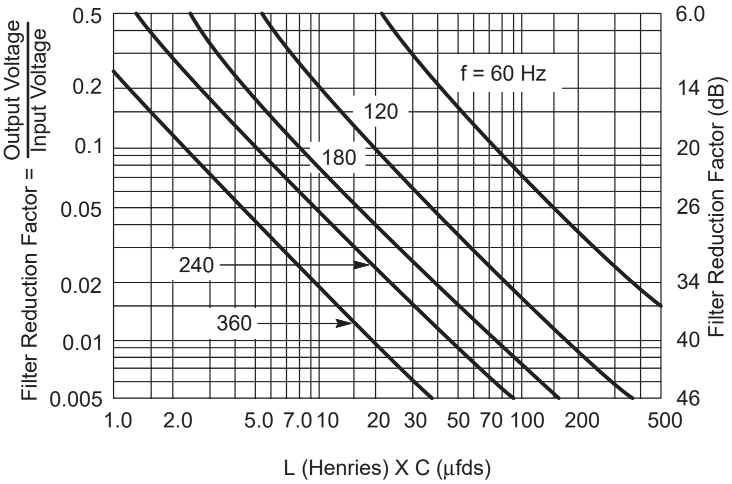


Figure 113. Reduction in Ripple Voltage Produced by a Single–Section Inductance–Capacitance Filter at Various Ripple Frequencies

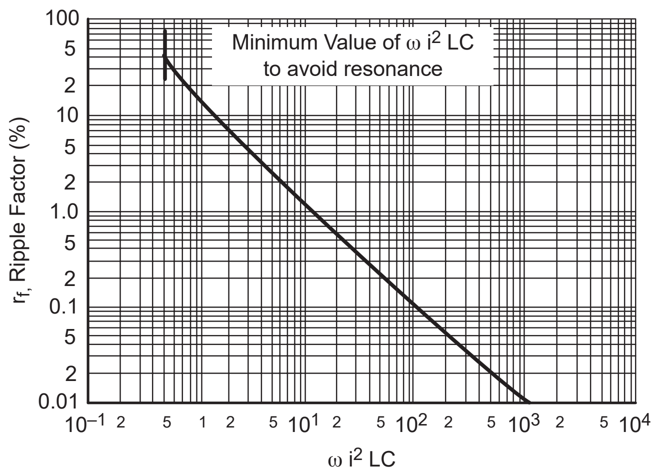


Figure 114. Ripple Factor for Single–Section *LC* Filter for Full–Wave Input

Graded Filters

When the output of a rectifier–filter system is called upon to supply voltages for several stages of an amplifier system, ordinarily the amount of ripple or hum voltage that can be tolerated is least for those stages that operate at the lowest signal–power levels. This makes it desirable to arrange the filter system so that the voltages applied to different circuits operated from the rectifier–filter system undergo different amounts of filtering.

For example, the output stage of an audio amplifier obtains its collector voltage directly from the input capacitor, which is permissible because of the high power level at which the output stage operates, combined with the hum–suppressing action of the usually used push–pull connection. Progressively increased filtering, is provided for the lower level stages, care being taken to design the system so that the reduction in ripple voltage introduced by the filter between stages is at least as great as the amplification of the stages.

A graded filter reduces to the lowest possible value the magnitude of the currents that must be carried by the series impedance arms of the filter and often makes it practical to use resistance–capacitance filter stages in parts of the system, as illustrated in Figure 114. This results in substantial economy in cost, weight, and size compared to an arrangement in which the entire rectifier output is subject to the maximum amount of filtering. A graded filter also provides isolation or decoupling between amplifier stages, thereby reducing regeneration.

Filter Component Selection

The inductors used in a low–frequency filter must have laminated iron cores with an air gap that is sufficient to prevent the dc magnetization from saturating the core. The inductance that is effective in the filter is the incremental inductance, which depends both upon the dc and the ac magnetizations of the core. In estimating the ac magnetization that can be expected, it is normally assumed that the alternating current flowing in the inductance is equal to the voltage of the lowest ripple frequency applied across the input of the filter section, divided by the reactance of the inductance of the section. The alternating magnetization in the inductance of the first section may be relatively large, whereas the alternating magnetization for the inductances of the other filter sections will be very small if the first section is at all effective.

The capacitors used in filters must be capable of continuously withstanding a dc voltage equal to the peak voltage applied to the rectifier. Electrolytic capacitors are ordinarily used where the peak voltages do not exceed 400 to 500 V. Such capacitors have very low cost in proportion to capacitance, but they possess the disadvantage of a limited life. Various types of plastic capacitors are usually used at higher voltages and also find use at lower voltages where long life is more important than low cost.

Example of Power Supply Design

Information given in this chapter is used to design a power supply for an audio amplifier in a home entertainment center. The dc supply voltage will be obtained from a 120 V, single–phase line. Electronic regulation is not necessary, economically feasible, or particularly desirable for such applications. The supply will consist of a step–down transformer, rectifier diodes, and a capacitive–input filter as shown in Figure 116.

Design Constraints

For this example, a supply is to be designed to meet the power requirements of a 20 W stereo amplifier (operating in a maximum ambient temperature of 55°C) which must drive 8–Ω speaker loads. The output power transistors are in a totem–pole configuration and operate from a single dc supply. Theoretically, a 36 V supply would be adequate, but a full load voltage of 40 V is required to make up for transistor and resistive losses. The average current drawn from both channels at rated power out (20 W per channel) is 1.43 A. To minimize distortion, idle current is 50 mA per channel. At zero signal, the ripple is critical and must be held to 400 mV maximum so that no hum is detectable. Regulation is not critical, but it is desirable to keep the no–load voltage below 50 V to ease the voltage requirements of the output transistors.

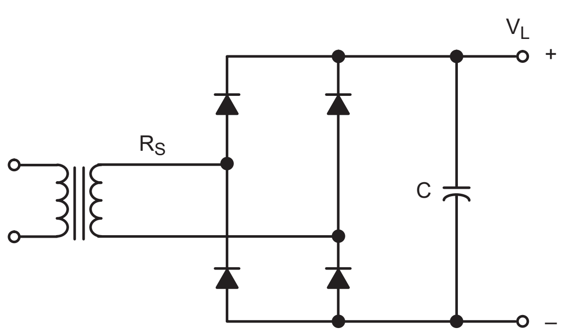


Figure 115. Bridge Circuit Used for Example Design

From the previous discussion, the power supply specifications are summarized in terms of the constants required:

1. $\omega = 2\pi f = 377$ rad/sec
2. $R_L = V_{L(DC)}/I_{L(DC)}$
   $= 40/1.43 = 28\ \Omega$ at full load
   $= 50/0.1 = 500\ \Omega$ at idle
3. $V_{L(DC)}$(full load)/$V_{L(DC)}$(idle) = 80%
4. r_f = 400 mV/50 V = 0.8%

Passive Component Selection

The choice of circuit is between a single–phase full–wave center–tap or a full–wave bridge circuit as outlined in Table 16 in Chapter 5. The center–tap requires only two diodes, but it has the disadvantages of requiring an additional winding, causing a poorer transformer utilization factor, and doubling the required voltage ratings of the diodes. The deciding factor in favor of using the bridge rectifier circuit is its higher secondary utilization factor. This is an important consideration when using a capacitive input filter because high rms currents are required from the transformer to obtain good output voltage regulation (i.e., a low R_S/R_L ratio is required).

The curves shown in Figure 110 are used to find a value for C to meet the ripple specification. To use the curves, it is necessary to estimate a value for the R_S/R_L ratio. This can be done by referring to Figure 108 and assuming that the idle dc voltage equals the peak of the input voltage and that the ωCR_L product is large enough to place operation in the right hand plateau of Figure 108. Based on these assumptions, note that to hold $V_{L(DC)}/V_M$ above 80%, R_S/R_L < 7%. If it is desired to minimize[1] C, then operation may move into the knee region and R_S/R_L may need to be made considerably less than 7% to hold $V_{L(DC)}/V_M$ to 80%.

From Figure 110, at ω_f = 0.8%, R_S/R_L = 1% (being conservative), read ωCR_L = 90. Therefore, C = 90/(377)(500) = 496 μF and a standard value of 500 μF can be used.

A suitable value for R_S to maintain the required voltage regulation can be obtained by using Figure 108. Under full load with C = 500 μF, ωCR_L = 5.2. Again assuming that the output voltage at idle equals the peak input voltage, which makes $V_{L(DC)}/V_M$ = 80%, read $R_S/R_L \approx 3.5\%$ at ωCR_L = 5.2. Therefore, $R_S = (0.035)(28) \approx 1.0\ \Omega$. This value may be largely composed of the sum of the transformer secondary resistance, the reflected primary resistance, and the diode dynamic resistance.

At idle, R_S/R_L = 1.0/500 = 0.2% and, as previously found, ωCR_L = 90. From Figure 108, $V_{L(DC)}/V_M$ is read as 97%, which verifies the validity of the assumption that at idle $V_{L(DC)} = V_M$.

[1]In many cases, minimizing C will result in lower cost. However, by choosing a larger value of C, R_S may be increased to obtain the same voltage regulation, with the beneficial effects of lower ruts and peak surge currents. The lower rms values may result in lower cost for the transformer and diodes, so that the overall power supply cost is less.

Transformer Selection

The peak output voltage developed across the capacitive filter has been specified as 50 V, which requires a transformer secondary rms voltage of 35 V. However, because of the forward voltage drops across the diodes, a standard 36 V transformer may be used. The transformer volt–ampere rating is dependent on rms current, which can be obtained from Figure 109. At full load $R_S/R_L = 3.5\%$ and $\omega CR_L = 5.2$. In full–wave circuits, n = 2, the value of $n\omega CR_L = 10.4$, and $R_S/R_L = 3.5\%$, yielding a diode current form factor of approximately 2.1. The load and secondary form factor, being full–wave, is $1/\sqrt{2}$ times the diode form factor. Consequently, the transformer rating is

$$VA = (36)\,(1.43)\,(2.1)/\sqrt{2} = 77.$$

Diode Selection

A diode bridge assembly is chosen for ease of installation. Since each diode's reverse voltage is clamped by the filter capacitor, the bridge voltage rating need only be sufficient to handle a high line condition, typically 130% of nominal. In this application, 100 V diodes are adequate.

In choosing a bridge, the total output current of 1.43 A is used as a selection guide rather than average current per diode. A series of bridges with 2.0 A current ratings will be evaluated for this application. It is necessary to determine whether these bridges can handle the repetitive peak currents into the filter and the initial surge current required to charge the filter capacitor.

The steady–state peak–to–average current ratio can be obtained from Figure 109. With $R_S/R_L = 3.5\%$ and $n\omega CR = 10.4$, a peak–to–average ratio per leg of 7 is interpolated. Consulting Figure 116, taken from the diode's data sheet, a peak–to–average ratio of 7 per leg yields a maximum full–wave average current capability of about 1.7 A at 55°C, which exceeds the requirement comfortably.

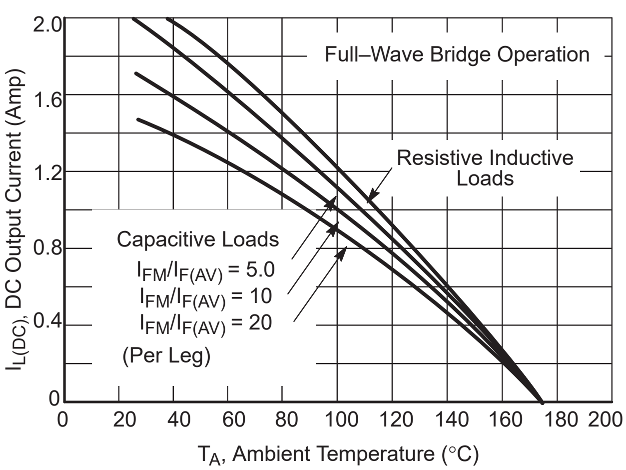


Figure 116. Derating Data for the Example

The diode bridge assembly has a surge current rating of 60 A for one cycle; i.e., it will handle two 60 A, half–cycle (8.3 ms) surges. The worst–case peak surge current in this application is

$$V_M/R_S = 50/1.0 = 50\text{ A}$$

The time of the surge is the time it will take to charge the capacitor, roughly the time constant of the series resistor and the filter capacitance, or

$$t \approx R_S C = (1.0)\,(500)\,\mu F = 0.5\text{ ms.}$$

Since the surge current is less than the rating and the time is much less than one cycle, the bridge will be satisfactory.

References

1. 1. J. Schaefer, *Rectifier Circuits,* John Wiley and Sons, Inc., New York, 1965, p. *258.*
2. 2. 0. H. Schade, *Analysis of Rectifier Operation,* Proc. IRE, Vol. 31, No. 7, July, 1943, pp. 343–344, 346–347.
3. 3. Robert H. Lee, *Origin and Characteristics of Harmonic Currents, Part I, Power Quality*, Volume 1, No. 5, 1990, pp. 349–351.
4. 4. Arthur W. Kelley and William F. Yadusky, *Rectifier Design for Minimum Line Current Harmonics and Maximum Power Factor,* Conference Proceedings of the Fourth Annual Power Electronics Conference and Exposition, Baltimore, 1989, IEEE Document number: 89Ch2719–3.

EQUATION 7. 1

is derived as follows:

$$\text{DC component} = \frac{V_M}{\pi}\int_{\omega t=0}^{\omega t=\pi} \sin \omega t \; d(\omega t) = \frac{2V_M}{\pi}$$

Ripple component of frequency $n\omega/2\pi$

$$= \frac{2V_M}{\pi}\int_{\omega t=0}^{\omega t=\pi} \cos\; n\omega t\; \sin\; \omega t \; d(\omega t)$$

$$= \frac{2V_M}{\pi}\left[\frac{\cos\,(n-1)\omega t}{2\,(n-1)} - \frac{\cos\,(n+1)\,\omega t}{2\,(n+1)}\right]_{\omega t = \pi}^{\omega t = 0}$$

$$= \frac{2V_M}{\pi}\left(\frac{-2}{n^2-1}\right)$$

In these equations *n* may have values 2, 4, etc.